ANNEX IV

Nonlinear wave-particle interaction I: anomalous diffusion

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Introduction. During this period, we studied the nonlinear interaction of relativistic electrons with a constant magnetic field and an oblique (a) monochromatic electromagnetic wave [1,2,3,4] (b) electromagnetic wavepacket of narrow bandwidth, consisting of a central frequency mode and two sidebands [5]. The dynamical behaviour of each system was visualized using Poincaré surfaces of sections (for (a) only) and energy distributions, over and under the estimated threshold to chaos. Issues related to the energetic, spatial and velocity diffusion across the ambient magnetic field lines were examined by following the evolution of the ensemble mean square displacements $\langle(\gamma-\gamma_0)^2\rangle$, $\langle(\mathbf{r}-\mathbf{r}_0)^2\rangle$ and $\langle(\mathbf{p}-\mathbf{p}_0)^2\rangle$ for various values of the wave power. We focused our attention in strong as well as moderate amplitudes, in the area near the threshold to chaos where the phase space is complex and a mixture of periodic and stochastic orbits co-exist. The type of diffusion in each space was determined and found to obey simple power law with scaling exponents indicant of sub-diffusion, a behavior connected with the existing regions of regular evolution in the phase space.

Hamiltonian formulation. Assuming the ambient plasma is cold, the normalized (over m_ec^2) Hamiltonian of the system (a) is $H = \gamma - p_z/n_z$ [2,3], where γ is the normalized (over m_ec^2) electron energy

$$\gamma = [1+(p_x+\varepsilon\cos\theta\sin\phi)^2+(p_y+x+\varepsilon\cos\phi)^2+(p_z-\varepsilon\sin\theta\sin\phi)^2]^{1/2} \qquad (1)$$

with p_x, p_y, p_z the normalized (over m_ec) canonical momenta, ε the normalized (over m_ec^2/e) wave amplitude, θ the wave propagation angle with respect to the z-axis and ϕ the wave phase, $\phi = \omega(n_xx + n_zz)$, with ω the normalized (over the cyclotron frequency) wave frequency and n_x, n_z the refraction index components. Similarly, the normalized Hamiltonian of the system (b) is of the form $H = \gamma - p_h/n_h$ [5], where

$$\gamma = [1+(p_x+\cos\theta\sum_{j=1}^{3}\varepsilon_j\sin\phi_j)^2+(p_y+x+\sum_{j=1}^{3}\varepsilon_j\cos\phi_j)^2+(p_a+p_h-\sin\theta\sum_{j=1}^{3}\varepsilon_j\sin\phi_j)^2]^{1/2} \qquad (2)$$

with p_x, p_y, p_a, p_h, the normalized canonical momenta, ε_j the normalized amplitude of the j-th wave mode (j = 1,2,3) and ϕ_j the phase of the j-th mode $\phi_j = \omega_j[n_{xj}x + (n_{zj} - n_h)z_a + n_hz_h]$, with ω_j the normalized j-th mode frequency, n_{xj}, n_{zj} the refraction index components and $1/n_h = \sum_{j=1}^{j=3}(1/n_{zj})$.

Threshold to chaos. For the system (a), it is known that significant chaos exists only for wave amplitudes larger than a critical value ε_{cr}, depending on the other system parameters [2,3]; this was found to stand also for case (b). This value provides the threshold of the wave power for the onset of chaos and all related effects, such as diffusion and acceleration. An estimate of ε_{cr} can be calculated by utilizing the fact that the electron acceleration comes together with the stochastic behaviour: by computing and visualizing the mean energy $\langle\gamma\rangle$ over an initially monoenergetic electron ensemble for many ε values, we expect at ε_{cr} a major burst to appear as acceleration occurs due to the onset of chaos. This is shown in fig.1, where we plot the mean electron energies granted during a motion time of T=3000 vs ε for both systems. In this and all the following cases, the initial energy of the N=1000 particles is taken γ_0=2.5, while the wave propagates at θ=40^0 with frequency $\omega = 6\pi$ MHz in the ambient field B_0=0.35G and plasma density $n_e = 10^2$ cm^{-3}; in (b), the packet extends ±0.02ω in the frequency space, with each mode having amplitude $\varepsilon/2$. The burst is seen to take place near ε_{cr} = 0.03 for (a) and ε_{cr} = 0.04 for (b), and that is the best estimate of the threshold value we can get using this method for the current system(s) parameters.

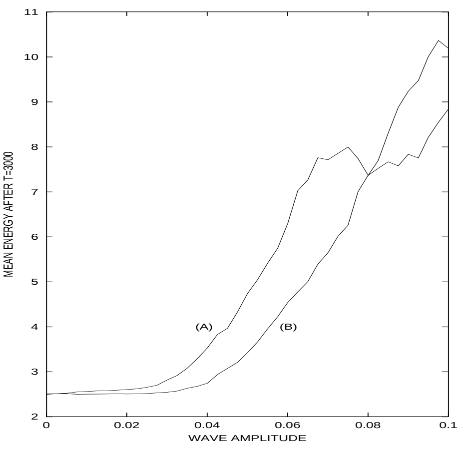

Figure 1. Ensemble mean energy <γ> vs ε for an orbit time of T=3000. N=1000 particles of initial energy γ_0=2.5 were used.

System dynamics. In the 2-degree-of-freedom case (a), the dynamics may be ascertained by using Poincare images of the phase space [2,3]. It turns up that chaotic phenomena prevail in all the phase plane except (i) an extended neighborhood of (0,0) covered by invariant curves and (ii) islands in the chaotic sea, corresponding to high-order resonances. Thus, the kind of motion depends strongly on the initial conditions. For (b), although mapping the phase space is not possible due to the 3-dimensions, the phase space topology is is expected to be similar. In both cases, the system dynamics can be seen from a statistical point of view, by forming the energy distribution of similar samples at several orbit times and parameters. An example of the results in both cases is in fig.2, where the distribution function at T=10^4 is plotted vs γ for ε =1. It is clear that the systems behave as stochastic, at least in the regions explored by the particles. The distribution has a canonical form, a sign that diffusion occurs, with almost all particles participating in the energy diffusion process. Acceleration is more intense for the electrons that interact with the wavepacket, as expected due to the increase of the electron-cyclotron resonances in the (b) case.

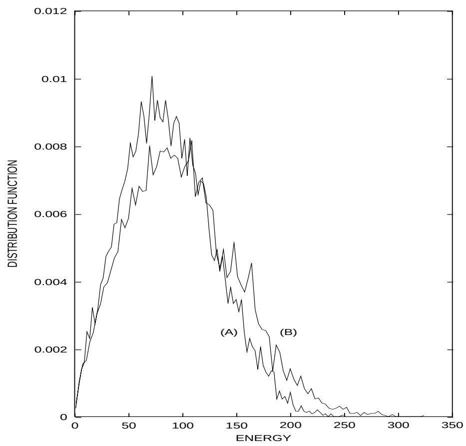

Figure 2. Energy distribution function f(γ) for wave amplitude ε = 1, after T=10000. The ensembles used consist of N=10000 particles with initial energy

Anomalous diffusion. The motion in phase spaces with chaos is rather complex, with the evolution in mostly stochastic areas being diffusive[2,4]. Diffusion may be viewed as a Brownian random walk, and this iscalled normal diffusion [6]. However, there are cases where diffusion is not normal due to the phase-space topology [6,7]. The diffusion typeis determined by the scaling in time of the mean square deviations; e.g.for energetic diffusion this reads $\langle(\gamma-\gamma_0)^2\rangle \propto t^{a_\gamma}$, where α_γ is the scaling exponent. Similar relations can be written also for the spatial and velocity diffusion,. If α_γ =1 then diffusion is normal, while if $\alpha_\gamma \neq 1$ we have anomalous diffusion, namely sub-diffusion for α_γ <1 and super-diffusion for α_γ <1 [6,7]. An example of applying these to (a),(b)is given in fig. 3, where the energy mean square deviations, as found from the orbits, are plotted vs time in log-log scales for ε = 0.5. In both cases, the behaviour is similar with the curves being linear after some time; thus, power laws exist for the displacements, at least after an elapsed time of motion. For the orbit part defined after this time, each exponent is the slope of the mean square deviation vs time log-log plot; thus, these exponents may be computed by linearly fitting the plots. In this fashion, we found the exponents in all spaces for ε within [0.05-1], fairly over ε_{cr}. Results for the case (a) are shown in fig.4, where the α's are plotted versus ε; the motion time was varied within [10^4,10^5], depending on the scaling variations due to the complexity of the phase-space. Evidently, for every $\varepsilon > \varepsilon_{cr}$ all exponents are less than 1; that means we have sub-diffusion. In more detail, for large ε the exponents are almost constant; as ε decreases, all α's start to decrease due to the enhancement of ordered structures in the phase space. The decrease becomes radical as the wave power reaches the threshold, below which the exponents are almost zero due to the lack of diffusion.

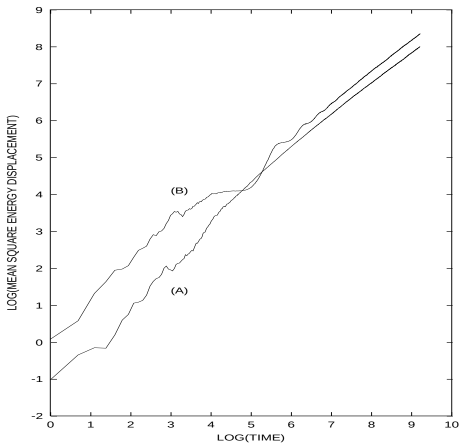

Figure 3. Log-log energy mean square displacements vs time for ε = 0.5. The ensembles used consist of N=10000 particles with initial energy γ_0=2.5.

Conclusion. During the study of the relativistic electron motion under packets of oblique electromagnetic waves in a uniform magnetic field, transport in phase space was found to scale with sub-diffusive rhythms. This abnormal diffusion is caused by the fractal sets of invariant tori existing in the system(s) phase space, which cause large time-space scaling of the particle kinetics and thus anomalous transport.

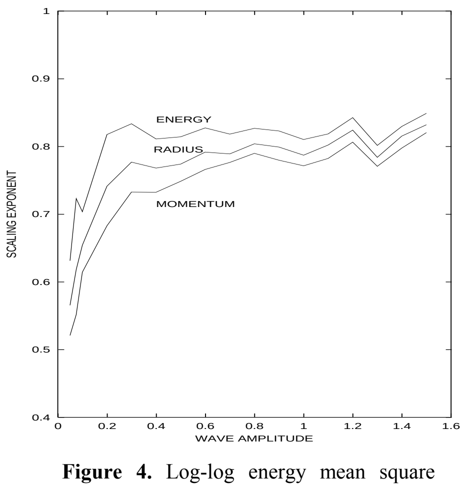

Figure 4. Log-log energy mean square displacements vs time for ε = 0.5. The ensembles used consist of N=10000 particles with initial energy γ_0=2.5.

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