ANNEX XIII

Two step control of resistive wall mode and the monodromy matrix

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In a recent publication [1] we proposed to stabilize the "resistive wall mode by modulating in time the resistivity of the wall. To study the stabilizing effect of this modulation we introduced the concept of an "effective potential" depending upon the instantaneous value of the resistivity. This led us to concentrate on the rather simple dissipative Mathieu-Hill equations, which resulted in very encouraging results related to the modulation strength of wall resistivity and the dissipation in the plasma.

The physical situation can be well understood by assuming viscous magnetohydrodynamics in the plasma, then a vacuum region inside a resistive wall with Ohm's law, and a vacuum region outside the wall extending to infinity or to a hypothetic perfectly conducting wall at a large but finite distance. To study Floquet stability of the actual equations (1) [2] of this system in real geometry of magnetic confinement systems is a formidable task, which should be preceded by a through investigation of simple and relevant models. Discretization of Eqs. (1) [2] allows, however, to define a monodromy matrix which is related to the change of the system due to the modulation in time over a period. Stability can be determined from its eigenvalues. In the "two-step" case the monodromy matrix is the product of two matrix exponentials:

$$M(T) = \exp\big((T - t_s)A_2\big)\exp(t_s A_1),$$

where A_1 *and* A_2 are time independent $n \times n$ matrices related to a matrix A depending periodically upon time in a piecewise constant manner, i.e., $A = A_1$ for $0 < t < t_5$ and $A = A_2$ for $t_5 < t < T$. In this case $M(t)$ can be evaluated either by reducing the matrix exponentials to polynomials on the eigenvalues of the corresponding matrices [on the basis of Cayley-Hamilton theorem (CH)] or, without knowledge of the eigenvalues, by using the Baker-Campbell-Hausdorff (BCH) formula. In the present work the two methods are discussed and applied to several examples including the "two-step" dissipative Hill's equation and a 3×3 system, a crude mock up of the actual system (Eqs. (22)-(24) [2]). It turns out that the CH and BCH methods are very powerful for instance, the first marginal curve for "negative energy" modes of the "two-step" Hill equation (13) [1] can be partly reproduced analytically. The 3×3 model, however, is too crude to give all qualitative features of the resistive wall mode, i.e. dynamical stabilization does not show up as expected because the "wall" has, so to say, zero thickness This means that we have to go to much larger systems approaching more realistically the plasma and a resistive wall having a thickness larger or comparable to the "skin" thickness. An accurate discretization of system (1) [1] with subsequent application of CH or BCH methods would lead to much higher numerical effort.

More details about this study can be found in Ref. [2]

1. *H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas **9**, 2662 (2002).*
2. *H. Tasso and G. N. Throumoulopoulos Phys. Lett. A, 307 304 (2003)*