ANNEX VIII

Chaotic electron dynamics in gyrotron resonators

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PHYSICAL MODEL

The equation which describes the electron motion in a gyrotron resonator has the following form [1]:

$$\frac{dp}{d\zeta} + i(\Delta + |p|^2 - 1)p = iFf(\zeta) \tag{1}$$

with the initial condition $p(\zeta_0)=\exp(i\theta_0)$, $0 \leq \theta_0 \leq 2\pi$. Here: p is the dimensionless transverse momentum of the electron, ζ is the dimensionless coordinate, Δ is the frequency mismatch, and F is the dimensionless beam to RF coupling factor. The differential equation (1) represents the cold-cavity approximation when the RF field $f(\zeta)$ depends only on the geometry of the resonator, but not on the electron motion (f does not depend on p). In this case the RF field can be approximated by a Gaussian $f(\zeta) = \exp\left[-\left(\frac{2\zeta}{\mu} - \sqrt{3}\right)^2\right]$, where $\mu = \pi(\beta_{\perp 0}^2 / \beta_{\parallel 0})L/\lambda$ is the dimensionless length of the resonator with length L. The aforementioned approximation is valid in resonators with high quality factors [2].

Considering as an unperturbed (integrable) system the one which describes the electron motion in the absence of the RF field, with Hamiltonian H_0 and transforming to action-angle variables (J,θ) [4] we obtain a Hamiltonian of the form: $H_0(J) = J^2 - \delta J$, where $\delta=1-\Delta$. The frequency of oscillations given by $\omega_\theta = dH_0(J)/dJ = 2J - \delta$. The Hamiltonian of the perturbed (near-integrable) system can be written in terms of the action-angle variables of the unperturbed system as follows:

$$H(J,\theta,\zeta;F) = H_0(J) + H_1(J,\theta,\zeta;F) \tag{2}$$

where

$$H_1(J,\theta,\zeta;F) = F\sqrt{2J}\ \mathrm{Im}\left\{\exp(i\theta) f(\zeta)\right\} \tag{3}$$

It has been, just recently, proved that the application of the KAM theory can be extended to Hamiltonian systems with aperiodic perturbations [5]. The approximate construction of the corresponding KAM curves can be made by using the Canonical Perturbation Theory [4] which utilizes successively near-identity canonical transformations to transform the Hamiltonian to a normal form in which the θ- and ζ- dependence is pushed to higher order terms with respect to the small parameter of the perturbation method. The approximate local invariant of the motion is given in terms of the new action

$$\bar{J} = J - F\sqrt{2J}\ \mathrm{Im}\left\{I_0 \exp(i\theta)\right\} \tag{4}$$

where

$$I_0(\omega_\theta(J),\zeta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{F(\Omega)}{\Omega + \omega_\theta} \exp(i\Omega\zeta)d\Omega \tag{5}$$

For the case of the Gaussian profile we have

$$I_0 = i\sigma\sqrt{\frac{\pi}{2}}e^{-\frac{\omega^2\sigma^2}{2}} e^{-i\omega(\zeta-\zeta_0)}\left[1 + \mathrm{erf}\left(\frac{(\zeta-\zeta_0) - i\omega_\theta\sigma^2}{\sqrt{2}\sigma}\right)\right]$$

where erf(x), $x \in$ C is the error function, $\sigma^2=\mu^2/8$ and $\zeta_0 = \sqrt{3}\mu/2$.

RESULTS AND DISCUSSION

In order to analyze the phase space of the system, we use appropriate Poincare surfaces of section. According to the KAM theorem as extended for the case of aperiodic time perturbations [5], some of these cylinders persist under small perturbations (small amplitude of the RF field), although the action is no longer a constant of the motion and the flow is not independent of the angle variable (loss of cylindrical symmetry). However, *under strong perturbation there are "cylinders" of a specific radius range which can be destroyed, resulting in a significant change in the topology of electron trajectories. As will be shown, the latter is essential for the operation of gyrotrons in parameter ranges of high efficiency.* The appropriate Poincare surface of section, defined as $\Sigma=\{(P,\zeta):Q=0,\ dQ/d\zeta>0\}$, can be used for tracking both clockwise and counterclockwise electron gyrations, which take place for $\delta>0$ and appropriate action values.

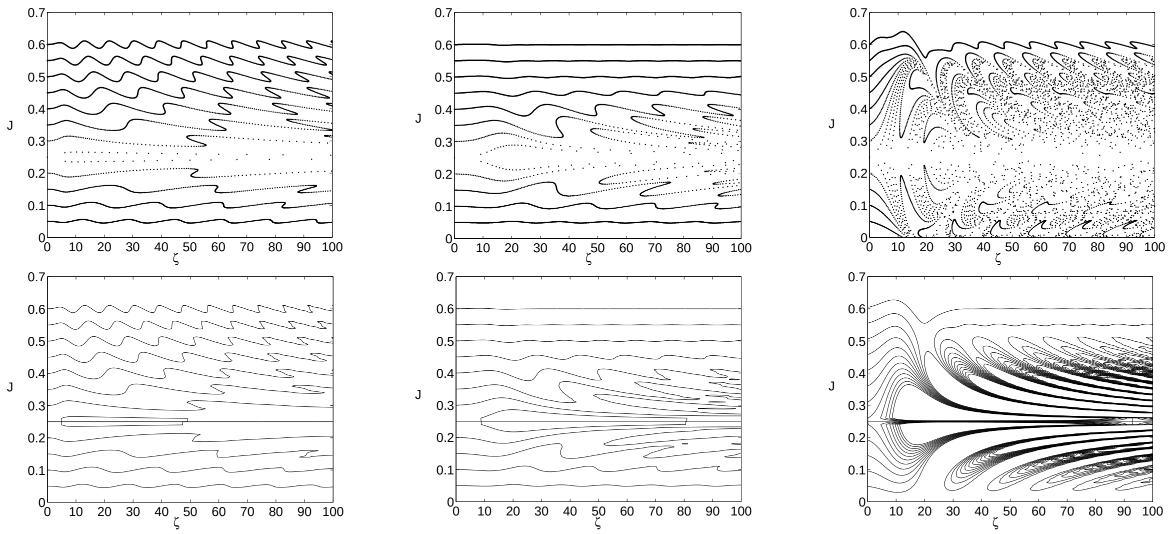

Fig. 1: Numerically (top) and analytically (bottom) obtained Poincare surfaces of section for $(F, \Delta, \mu)=(0.005, 0.5, 5)$, $(0.005, 0.5, 15)$ and $(0.125, 0.5, 17)$ (left to right).

In Fig. 1 several cases of Poincare surfaces of section are shown, exhibiting the features of electron dynamics under a variety of gyrotron operating parameters F, μ and Δ. For all cases there exists an area of drastic interactions around $J_0=\delta/2$, which is the boundary action value between areas of opposite direction of rotation with respect to the frame of the RF field, for the unperturbed system. It is obvious that μ determines the width of the strong interaction area around J_0, for a given F. A small μ describes a short RF profile in ζ, which has a more spread frequency spectrum and consequently can resonate with a greater range of electron frequencies (and actions), while the opposite holds for a large μ. In the strong interaction area, electrons with initial actions $J>J_0$ can move to areas of significantly smaller actions $J<J_0$, after undergoing a change of the direction of rotation in the frame of the RF field and evolve in ζ with an almost constant J after a transition length. *This transition from a high action value to a lower one is the essential mechanism of efficient gyrotron operation, and is strongly nonuniform in the electron initial angle*. The chaotic dependence on the initial angle has also been studied in terms of the Lyapunov exponents, measuring the divergence of nearby electron trajectories, in [6], where the action reduction accompanied with inversion of the rotation direction has also been shown. The central action value of the area of strong interaction is determined by the frequency mismatch. Specific values of Δ combined with appropriate values of F and μ can result in lower boundaries of the action, under which an initially large action cannot be reduced. The latter is of practical importance for gyrotron operation since knowledge of the energy (action) distribution of electrons after their interaction with the RF field in the resonator is a prerequisite for optimizing depressed collectors.

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