ANNEX XVII

Generation and saturation of large scale flows in flute turbulence

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Electrostatic turbulence, driven by spatial gradients, is believed to be the dominant source of anomalous transport in magnetically confined fusion plasmas. Special emphasis has been given lately on the properties of large scale anisotropic flows generated by the drift–type turbulence, due to the critical role they play in the regulation of the low–frequency drift instabilities and consequently of the levels of turbulent transport. The high plasma confinement modes are attributed to the presence of large scale poloidal flows (zonal flows). Streamers, on the other hand, are ineffective at inhibiting radial transport and, due to their long radial correlation length, may lead to enhanced or bursty levels of transport. The aim of the present work is the numerical investigation of the generation and saturation of large scale anisotropic flows by an evolving magnetic–curvature–driven flute instability. Flute modes are low–frequency ($\omega \ll \omega_{ci}$) electrostatic oscillations of a non–uniform magnetoplasma which are elongated along the magnetic field $k_{\parallel} = 0$ (flute limit) and become unstable due to the combined effects of the density inhomogeneity and the curvature of the magnetic field lines.

We will focus on the description of the excitation, interaction and suppression of the largest–scale anisotropic modes, i.e. the zonal and the streamer modes. Zonal modes are defined here as modes with $k_y = 0$ and a small but finite radial scale lengths k_x^{-1}. Streamer modes are defined as the modes with $k_x = 0$ and small but finite poloidal wavenumbers k_y. A weakly inhomogeneous magnetized plasma with characteristic inhomogeneity scale length L_n along the radial axis x is considered. The magnitude $B(x)$ and the unit vector $\mathbf{b}$ of the curved magnetic field are modeled by $B(x) = B_0\left(1 - x/R\right)$ and $\mathbf{b} = \hat{z} - (z/R)\hat{x}$, respectively, where R ($> L_n$) is the curvature radius of the magnetic field lines. Starting from the two–fluid plasma equations, and assuming flute–type ($k_{\parallel} = 0$), quasi-neutral, electrostatic oscillations, it is found that the magnetic–curvature–driven flute modes are described by the following set of dimensionless coupled equations for the perturbed electrostatic potential ϕ and density n:

$$\left(\partial_t - \tau v_n \partial_y\right)\nabla_\perp^2\phi + v_g(1+\tau)\partial_y n = \tau \mathrm{div}\left\{\nabla_\perp\phi, n\right\} + \left\{\nabla_\perp^2\phi, \phi\right\} + \mu\nabla^4\left(\phi + \tau n\right) \quad (1)$$

$$\left(\partial_t + v_g\partial_y\right)n + (v_n - v_g)\partial_y\phi = \left\{n, \phi\right\} + D\nabla^2 n, \quad (2)$$

where $\{f, g\} = \hat{z}\times\nabla f\cdot\nabla g$. In Eqs. (1,2), the electrostatic potential has been normalized by T_e/e, the time by the ion cyclotron frequency ω_{ci}, the lengths by the ion Larmor radius $\rho = c_s/\omega_{ci}$ defined at the electron temperature (here $c_s^2 = T_e/m_i$), the density by the unperturbed plasma density n_0, and the temperatures by the electron temperature T_e. The ion temperature is now denoted by $\tau\ (= T_i/T_e)$. In dimensional units, the electron curvature and diamagnetic drift velocities are given by $v_g = 2c_s^2/(R\omega_{ci})$ and $v_n = c_s^2/(L_n\omega_{ci})$, while the viscous and diffusion coefficients μ and D are given by $\mu = (3/10)(T_i\nu_i/\omega_{ci}m_i)$, $D = m_eT_e\nu_e/(eB)^2$, where ν_j denotes the collision frequency of the plasma particles ($j = i, e$). We linearize Eqs. (1,2) and determine the frequency $\omega_k = \frac{1}{2}\left[k_y(v_g - \tau v_n)\right]$, and the growth rate

$$\gamma_k = \frac{1}{2}\left[\sqrt{\frac{4k_y^2}{k_\perp^2}v_g(v_n - v_g)(1+\tau) - k_y^2(\tau v_n + v_g)^2 + k_\perp^4(D-\mu)^2} - k_\perp^2(D+\mu)\right] \quad (3)$$

of the unstable flute modes. The presence of finite ion temperature leads to a decrease of the frequency and subsequently of the characteristic wave velocities of the flute modes. For $\tau < v_g/v_n$ the flute modes propagate in the same direction with the electron curvature drift velocity, while for $\tau > v_g/v_n$ it propagates in the opposite direction. Furthermore, for finite ion temperature τ, the growth rate of the most unstable flute modes increases due to the ion curvature drift, while the spectrum of the unstable flute modes gets narrower due to the stabilization of the short wavelength modes by the ion diamagnetic drift. In the temporal evolution of the flute instability, the inverse energy cascade may lead to the generation of large scale flows. As one may see from Eqs. (1,2), there exist several non–linear terms which determine the cascading properties of the flute turbulence. The polarization drift non–linearity $\left\{\nabla_{\perp}^{2}\phi,\phi\right\}$ is responsible for the energy cascading towards large scale flows, while the convective non–linearity $\left\{n,\phi\right\}$ is known to cascade energy towards short scales. Moreover, the diamagnetic component of the polarization drift non–linearity $\tau\mathrm{div}\left\{\nabla_{\perp}\phi,n\right\}$, which is attributed to the finite ion Larmor radius, is expected to lead to direct cascading of the fluctuation energy towards short scales.

DYNAMICS OF FLUTE TURBULENCE

We have studied numerically the temporal evolution of the system described by Eqs. (1, 2) and the subsequent excitation of large scale flows by using a dealized pseudospectral code in a numerical grid of 128 × 128 points. The marching in time is performed with a fourth order Runge–Kutta technique with adaptive step–size. We have imposed periodic boundary conditions and considered a physical domain in the xy plane of area $\Delta x\times\Delta y=[(-30\pi,30\pi)\times(-30\pi,30\pi)]$. The minimum finite wavenumber which can be resolved with our scheme is $k_0=k_{x0}=k_{y0}=0.033$. In the numerical simulations, we have chosen the normalized (over the sound velocity) values of the electron diamagnetic and the curvature drift velocities to be $v_n=0.03$ and $v_g=0.01$, respectively, while the viscosity and the diffusion coefficients are fixed at $D=\mu=0.1$. The initial conditions for the potential and the density perturbations consist of an *isotropic spectrum* of small amplitude and randomly phased Fourier modes.

In the presentation of the numerical results which follows, we focus on the description of the three major distinct phases associated with the evolution of the dominant large–scale flute modes.
The first phase of the evolution of the flute instabilities is characterized by the growth of the linearly unstable flute modes. During this phase, patterns of radial streamers of the fluctuating potential and density are formed in the real space, since for given finite poloidal wave number k_y the flute modes of maximum growth rate are those of $k_x=0$, i.e. the streamer modes. The growth of the streamer modes continues until a suppression mechanism sets on. The duration of the growth is different for each streamer mode as the onset of the suppression depends on the mode's wavenumber. In general, the smaller the streamer mode is, the faster, and consequently at smaller amplitudes, it gets suppressed (see Fig. 1a).

When the amplitude of the flute perturbations reaches a critical value, zonal modes are excited. This is a purely non–linear effect and arises due to the non–linear coupling of the linearly grown flute modes. In Fig. 1b, the excitation and the evolution of the three largest zonal modes is depicted. These modes are generated almost simultaneously (around $t=180$) and grow exponentially with similar growth rates. The growth rate of the most unstable zonal mode (the one with $\vec{k}=(k_0,0)$) has approximately double the value of that of the dominant linear instability (that with $\vec{k}=(0,k_0)$). This is due to the quadratic nature of the nonlinear terms in Eqs. (1,2) and indicates that the zonal modes grow under the action of at least a couple of linearly amplified flute modes of small but finite poloidal wave number.
When the potential amplitude of the dominant non–linearly growing zonal mode becomes of order similar to the potential amplitude of the most grown streamer mode, both amplitudes start to oscillate in an out–of phase manner fashion. The result of this mode coupling is the suppression of the streamer instability and the saturation of the growth of the zonal mode. Similar description accounts also for the evolution and suppression of the smaller zonal and streamer modes (cf. Fig. 1), since the mechanism is qualitatively the same. The linear flute–instabilities, as the streamer modes, are suppressed through the shear stabilization mechanism $k_x^2\Phi$, which is provided by the growing zonal modes. This leads to the subsequent saturation of the zonal modes as well, since the flute modes which were responsible for the zonal growth. In order to shed some light onto the coupling mechanism between the dominant anisotropic modes, we have investigated the role of the largest and most grown isotropic modes $\vec{k}=(\pm k_0,\pm k_0)$ which provide the necessary matching conditions for a three-wave coupling between the largest anisotropic modes (see Fig 2). A series of numerical results show that the large–scale isotropic modes, being the coupling carrier between the largest anisotropic modes, support the mechanism: a) for the further growth of the zonal mode, b) for the suppression of the largest streamer mode and hence, c) for the subsequent formation of the poloidal flow. From the above, it is evident that short scale fluctuations can be significant for the generation of zonal flows, and large scale isotropic modes for the suppression of the flute instabilities. Furthermore the modes of the potential

and the density are organized differently on reaching the dynamical equilibrium that determines the saturated state of the flute turbulence. This result is not surprising since the plasma density response is not Boltzmannian in the flute limit, and hence the density fluctuations are expected to behave differently than the potential ones.

Further numerical studies we made on the role of $\tau \mathrm{div}\{\nabla_{\perp}\phi, n\}$ showed that the diamagnetic component of the polarization drift non–linearity, a) suppresses the inverse cascade towards large scale modes, and b) stabilizes the secondary excitations of streamers.

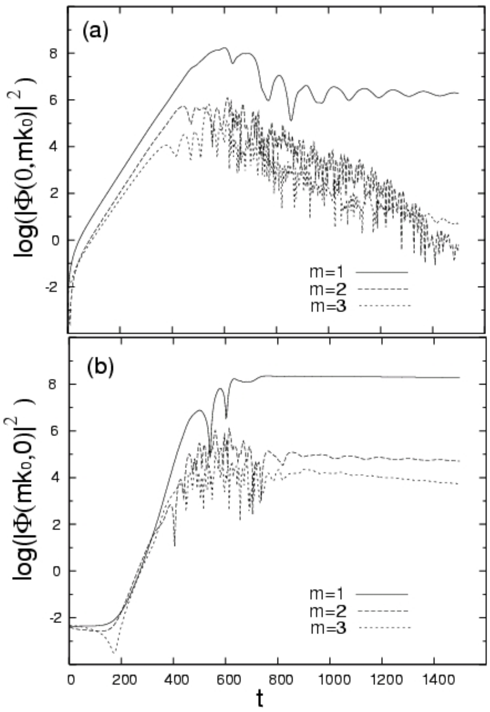

Fig. 1 Evolution and saturation of the largest streamer (upper panel) and zonal modes**]** (lower panel) of the potential. The shorter zonal and streamer mode instabilities saturate earlier and at lower amplitudes compared to the largest ones. In the saturated state, the amplitudes of the zonal modes and the largest streamer mode remain constant, while the smaller streamer modes get damped.

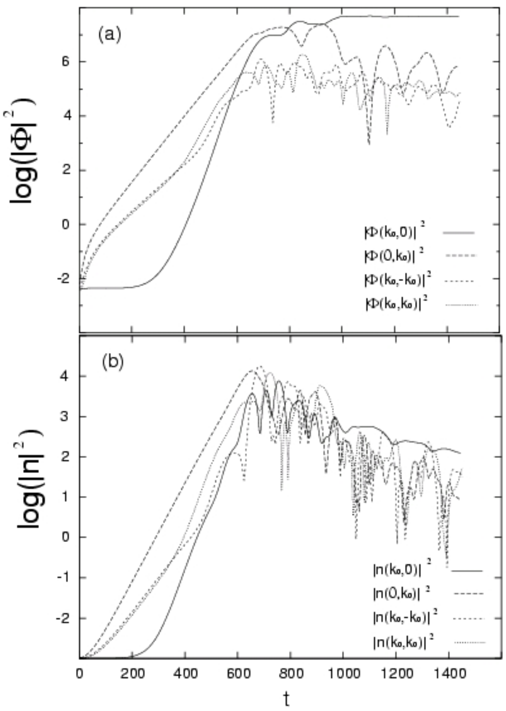

Fig. 2 Temporal evolution and saturation of the dominant isotropic and anisotropic modes for (a) the potential and (b) the density in the cold ion limit. The amplitudes of the isotropic modes in the saturation state are significant for the density fluctuations

SUMMARY

We have numerically investigated the excitation and suppression of large scale anisotropic modes as a result of the development of the flute instability. The initial formation of the streamer flow is attributed to the linear growth of the streamer modes, while the subsequent formation of the zonal flow is the result of the excitation of large–scale zonal modes through the inverse energy cascading mechanism. The most grown instabilities are the largest–scale ones and saturate last. The numerical results show that their suppression can be attributed to the non–linear interaction between the largest scale flute modes. The saturated state which follows is characterized by the domination of the largest zonal mode for the potential. However, the complexity increases when ion temperature effects are considered. It was also shown that modes of various scales are rather significant for the suppression of the flute instabilities. Hence, theoretical models based on the scale separation approximation, or zero models which incorporate only short scale fluctuations, STs and ZFs, may only be adequate to describe just the excitation of the zonal modes.

REFERENCE

I. Sandberg *et al.* Phys. Plasmas **12**, 032503 (2005) and references therein.