ANNEX XII

Scattered-field FDTD algorithm for hot anisotropic plasma with applications to EC heating*

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INTRODUCTION

The injection of electron-cyclotron (EC) waves is a standard method for coupling energy to plasma electrons in modern fusion devices (tokamaks, stellarators), with primary applications the plasma heating (ECRH) and the noninductive current drive (ECCD) (see [1] and references therein). In fusion experiments, the EC waves are launched in the plasma in the form of spatially narrow beams, and interact with the electrons when the EC resonance condition is fulfilled

$$\omega - k_{\|}v_{\|} - \frac{l\omega_c}{\gamma} = 0 \qquad (l = \pm 1, \pm 2, \ldots), \tag{1}$$

where ω is the wave frequency, $\omega_c = eB_0/m_e$ is the cyclotron frequency (B_0 is the magnetic field), $k_{\|}$, $v_{\|}$ are the wave number and the electron velocity components parallel to the magnetic field, and γ is the Lorentz factor. Since the cyclotron frequency is proportional to the magnetic field, which is not uniform in fusion devices, this condition is satisfied in a narrow spatial region, called the resonance layer.

The propagation of EC waves in the plasma is described by Maxwell's equations. In general, to obtain a full solution to the problem is burdensome because these equations are partial differential equations (PDE). In numerical applications, a PDE is equivalent to a spatial grid progressing on a time grid, which for the wave and plasma parameters occurring in fusion may be very resource-demanding; in some cases prohibitively. When the wavelength is small compared to the scale length of inhomogeneity of the plasma, a simplification is reached by frequency-domain asymptotic methods: ray tracing [2], quasi-optics [3] or beam tracing [4]. The solution is obtained over Hamiltonian differential equations, where the dispersion function plays the role of the Hamiltonian. The plasma response is derived in terms of the linear theory of plasma oscillations [5], and based on that, for typical experimental parameters, the wave intensity is small and falls in the linear regime.

The Finite Difference Time Domain method (FDTD) is nowadays recognised as a reliable tool in numerical electromagnetism. Among its strengths, it enables the direct visualisation of the electromagnetic fields inside the medium, it allows specifying the material at all points within the computational domain, and by using efficient boundary conditions it simulates propagation beyond the limits of the problem space. However, since FDTD requires the entire domain to be gridded, with the grids being smaller than the wavelength, to obtain results may require very large domains and long runtimes. For realistic ECRH simulations, the required spatial resolution makes the computational needs extremely demanding, and that is why there has been little application of FDTD to those problems up to day.

GENERAL ASPECTS OF FDTD

The FDTD method is a simple and elegant way to discretise the differential form of Maxwell's equations. In terms of the finite-difference approximation, the basic step for the solution is the setup of the spatial and temporal grids on which the discrete equations will progress numerically. In 1966, Yee introduced the pioneer FDTD scheme for the case of isotropic lossless material [6]. To obtain the fields throughout the computational domain, Yee used electric and magnetic field grids which are spatially and temporally offset. The placement of the fields is such that the space is filled with interlinked contours where the laws of Faraday and Ampere (curl equations) may be applied, and also the divergence equations are implicitly enforced. To proceed in time, a leapfrog scheme is implemented: **E** is computed and stored in a time-point using the **H**-data from the previous point, and the same goes for **H**.

SCATTERED-FIELD FDTD FORMALISM FOR ANISOTROPIC PLASMA

We are interested in applying FDTD, as a wave solver, to the modelling of EC heating. The fusion plasma, as a medium, may be assumed stationary and linear; however it is anisotropic due to the magnetic field. Therefore, it is necessary to generalise the Yee formulation in order to treat anisotropic materials. Mathematically, this means the introduction of tensor, instead of scalar, plasma response. The total-field

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formalism for an anisotropic medium has been derived by Schneider and Hudson [7]. In their paper, FDTD is extended to accommodate nonzero off-diagonal elements in the permittivity and conductivity, and the resulting equations differ substantially from the ones of Yee. Following this spirit, we present the scattered-field formulation for the problem.

NUMERICAL APPLICATION

We present a numerical application to the perpendicular propagation of an EC beam in simplified tokamak geometry (slab). The simplicity lies in adopting $r_p << r_t$, where r_t, r_p are the toroidal and poloidal radii. This assumption, known as the large aspect ratio approximation, corresponds to a flat (rather than curved) magnetic geometry with negligent poloidal field. As a consequence, the plasma properties (magnetic field, density and temperature) may be assumed to vary only along the x-axis, in the region $[-r_p, r_p]$ and the external magnetic field to lie just along z. The magnetic field increases from the one plasma edge $x = r_p$ (low-field side) to the other $x = -r_p$ (high-field side), while the density and temperature rise from the edges to the centre of the plasma $x = 0$.

We explored first the FDTD simulations of wave propagation in cold plasma. For propagation under the warm plasma model, the plasma response is almost identical with the cold plasma. This is because the only difference between the cold and warm plasma dielectric tensors is a finite pressure term, coming from the lowest-order thermal correction, which however does not describe the collisionless damping owed to resonant electrons. Without collisions, both the cold and warm plasma tensors are Hermitian and, consequently, contain no wave dissipation. For modelling collisionless absorption, the detailed treatment provided by kinetic theory is necessary.

In *Fig. 1* the result of a full-wave solution is compared to a beam tracing asymptotic solution. One can see that, despite the difference in the physical model, the agreement between the results of the two codes is very good. This happens because, in the plain slab model, absorption is described entirely by the anti-Hermitian part of the dielectric tensor. Consequently, the power deposition as a function of x becomes completely independent of the details of the beam trajectory. In particular, the optical depth of the plasma slab is not changed by the effects of kinetic dispersion on beam propagation (see also [6]).

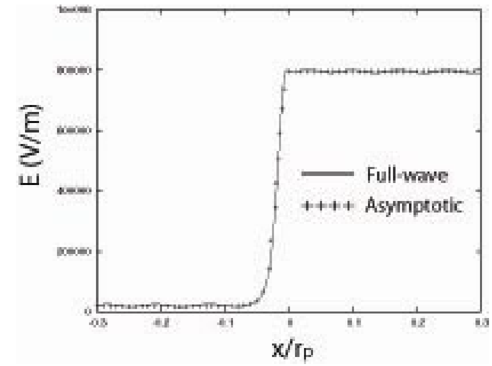

Fig. 1: Comparison of the full-wave solution and the beam-tracing asymptotic solution.

CONCLUSIONS-DISCUSSION

We present a full-wave model of EC propagation in hot anisotropic plasma, based on the FDTD method. For this type of problem, frequency-domain asymptotic methods are most popular, because of the ability to derive analytic expressions, in some cases even exact, for the plasma response. For a constant-frequency wave propagating in stationary plasma (the most common approach to ECRH), FDTD can be applied directly using the dielectric response of the plasma as derived in the frequency domain. The scattered-field formulation for anisotropic plasma is derived in compact form, as a continuation of [7]. A numerical application is presented in slab geometry, allowing a simpler description of the plasma-wave coupling, based on the 1D implementation of FDTD. The code ECFW studies the phenomena under different physics models for the dielectric plasma response.

The main numerical results are summarised as follows: In all cases, the fields reach a steady-state after a characteristic time, as expected due to the lack of time dependence in the plasma response. For cold/warm plasma, the electric field amplitude remains constant along propagation, because these models do not include the effect of cyclotron absorption. In hot plasma, the absorption of the wave is found to occur in a very narrow region, known as the resonance layer, which is defined by the wave frequency and the magnetic field profile. Specifically, the position of the resonance layer on the x-axis is controllable, i.e. it can be shifted towards the low- or the high-field side, by changing either the wave frequency or the magnetic field on the axis. The generated current is also localised, practically only inside the resonance layer. Benchmarking of ECFW with a code based on the beam tracing asymptotic method proved successful.

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