ANNEX 17

On the effect of the approximations used in gyrotron interaction calculations*

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INTRODUCTION

In a gyrotron, a helical electron beam, formed by a MIG-type electron gun and guided by an external magnetostatic field, delivers energy to a RF electromagnetic wave (i. e. a TE mode supported by the gyrotron cavity) through electron cyclotron resonance. Numerical simulations of the beam-wave interaction in high-power gyrotrons are the basic tool for the design of the interaction cavity in these devices. In the interaction codes several assumptions are usually made in order to achieve fast simulations, which is a necessary feature of a designing tool. However, as the resonators get larger to meet the increasing needs in output power and, consequently, their mode spectrum becomes denser, the validity of some of the aforementioned assumptions needs to be revisited. We investigate the influence of these assumptions on the electron motion using the pertinent numerical codes in the code-packages EURIDICE and *Ariadne*++.

The interaction code in the package EURIDICE [1] is based on the slow-time-scale approximation [2]. In this approximation, which, as a rule, is employed in similar interaction codes used for gyrotron design ([3], [4]), the equations of the interaction are averaged over the gyro-period, on the grounds that the quantities of interest do not vary much during a gyro-period. In this way, significant computation time is saved since the time-step of the integration of the equations can be larger than the gyro-period. In addition, further simplifying assumptions are used to achieve faster calculations. Contrary to EURIDICE, the routine for calculating the electron motion in a given RF field in the code *Ariadne*++ [5] uses no approximations. By the comparison of the results of the two codes, we evaluate the influence of each approximation used in EURIDICE on the electron trajectories for a case relevant to the European 170 GHz coaxial gyrotron for ITER [6]. In this way we make a first step on revisiting the validity of the usual assumptions of interaction codes for the case of the present high-power gyrotrons.

SLOW-VARIABLES MODEL AND EMPLOYED ASSUMPTIONS

We consider a single TE_{mp} mode in a coaxial gyrotron cavity with corrugated insert, whose electric and magnetic fields in the region of the electron beam are given by [7]

$$\mathbf{E}_{RF} = V_{\max} f(z)\mathbf{e}_{mp}\left(R,\Phi;z\right)e^{i\omega t}\,, \quad \mathbf{B}_{RF} = \frac{i}{\omega}\nabla\times\mathbf{E}_{RF} \tag{1}$$

$$\mathbf{e}_{mp} = k_{\perp}C_{mp}\left[\frac{im}{k_{\perp}R}Z_{mp}(k_{\perp}R)\hat{\mathbf{R}} + Z'_{mp}(k_{\perp}R)\hat{\mathbf{\Phi}}\right]e^{-im\Phi} \tag{2}$$

In this expression for the electromagnetic field, (R, Φ, z) are cylindrical coordinates around the cavity axis z, m and p are integers denoting the mode order, ω is the mode angular frequency, the constant $V_{\max}$ (in Volts) denotes the mode amplitude, and the complex function $f(z)$ describes the field profile along the cavity axis. In the expression of the eigenvector $\mathbf{e}_{mp}$, $k_{\perp}$ is the transverse wave number, C_{mp} is a normalisation factor, and the cylindrical function Z_{mp} is a linear combination of Bessel functions of the first and second kind. These three quantities are mild functions of z, due to the mild variation of the geometric characteristics of the coaxial gyrotron cavity along z, and their expressions can be found in [7].

The equation of the electron motion in the gyrotron cavity in the presence of a TE_{mp} mode, together with a strong uniform axial magnetostatic field $\mathbf{B}_0$ and also an electric field $\mathbf{E}_0$, due to the imposed high voltage and to the beam self-field, is

$$\frac{d\mathbf{p}}{dt} = -e\left[\left(\mathbf{E}_0 + \mathrm{Re}\{\mathbf{E}_{RF}\}\right) + \frac{\mathbf{p}}{\gamma m_e}\times\left(\mathbf{B}_0 + \mathrm{Re}\{\mathbf{B}_{RF}\}\right)\right] \tag{3}$$

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where **p** is the electron momentum, γ is the relativistic factor and e, m_e are the absolute electron charge and rest mass, respectively. After a number of assumptions and considerable manipulation, the following gyro-averaged equation of electron motion can be obtained:

$$\frac{dp_\perp}{d\zeta} + ip_\perp\left(\gamma - \gamma_0\right) = -\frac{\gamma}{2}\frac{V_{\max}}{V_0} f(\zeta)\frac{ck_\perp}{\omega_c} C_{mp} Z_{(m-1)p}\left(k_\perp R_e\right) e^{i\left(\frac{\omega}{\omega_c}-1\right)\gamma_z\zeta} \tag{4}$$

Here, $p_\perp = u_\perp \exp(-i\varphi)$ is the complex transverse momentum of an electron with normalised transverse momentum $u_\perp = \gamma v_\perp/c$, and $\zeta = [\omega_c/(cu_{\|})]z$ is the normalised axial coordinate. ω_c, $u_{\|}$ and γ_0 are the initial values of the relativistic electron cyclotron frequency, the normalised axial electron momentum, and the relativistic factor, respectively. In addition, $V_0 = 511$ kV and R_e is the electron beam radius. Equation (4) refers to an electron in resonance with the TE mode at the fundamental cyclotron frequency and the initial condition is $p_\perp(\zeta = 0) = u_{\perp 0}\exp(-i\varphi_0)$, with φ_0 uniformly distributed in $[0, 2\pi)$. This equation is similar, and under some assumptions equivalent, to the equations presented in [2] and to those used in the codes of [3]-[4]. The basic assumptions used to obtain (4) from (3) are:

(i) The electric field $\mathbf{E}_0$ is neglected.

(ii) The electron is in resonance with the TE mode and this makes possible to neglect fast-varying terms by taking an average of (4) over the gyro-period $2\pi/\omega_c$. The quantities in (4) are thus slow variables, representing the mean value of the corresponding original fast-varying quantities.

(iii) As a consequence of (i)-(ii), the position (R_e, Φ_e) of the electron guiding centre remains unchanged.

(iv) The TE mode is close to cutoff, hence the transverse components of $\mathbf{B}_{RF}$ are neglected. [This assumption combined with (i) makes the axial momentum a constant of the motion.]

(v) It is assumed that $v_\perp/c << 1$, because the beam is weakly relativistic. It turns out that with this assumption, (v1) the axial component of $\mathbf{B}_{RF}$ can be neglected and (v2) the Bessel functions containing the Larmor radius in the argument can be approximated by the first term of their small argument expansion.

(vi) The axial electron velocity $v_{\|}$ is also assumed constant, on the grounds of the conserved parallel momentum and the small change in γ.

COMPARISONS AND DISCUSSION

The interaction routine in the code *Ariadne*++ solves directly (3). The interaction code in the code-package EURIDICE is a slow-variables code solving (4), however it has the option for dropping the assumptions (v)-(vi). We compared the results of the two codes for the nominal operating parameters of the European gyrotron for ITER [6], that is, beam voltage V_b =90 kV, electron velocity ratio $\alpha = 1.3$, $B_0 = 6.86$ T (resulting in $\gamma_0 = 1.176$, $u_{\perp 0} = 0.491$, $\omega_c/(2\pi) = 163.526$ GHz), and $R_e = 10$ mm. The purpose of the comparison was to assess the influence of the approximations used in (4) on the electron motion and on the electron efficiency. For simplicity, the cold-cavity field profile $f(z)$ of the operating $TE_{34,19}$ mode was used, assuming an output power of 2 MW ($V_{\max} = 202$ kV) at the cold-cavity frequency of 169.99 GHz. We let a set of 36 electrons (having 36 different initial phases φ_0 uniformly distributed) to pass once through the cavity with the fields fixed as described before, and we calculated the electron efficiency as $\eta_{\text{elec}} = (\gamma_0 - \langle\gamma_{\text{out}}\rangle)/(\gamma_0 - 1)$, with $\langle\gamma_{\text{out}}\rangle$ the mean value at the exit.

First, the assumptions (i) and (iv) were introduced in *Ariadne*++ and, at the same time, the assumptions (v) and (vi) were dropped from EURIDICE. In this way, the codes differ only in (ii) and (iii), and the comparison can assess the validity of the slow variables approximation (gyro-averaging). In *Fig. 1*, the evolution of the electron momentum through the cavity is shown for three representative initial phases. It can be seen that the agreement of the codes, as far as the mean values are concerned, is very good and the information the slow-variables model primarily misses is, as expected, the small fluctuations around the mean due to the gyro-motion. This is also evident in the calculated efficiency, as shown in Table I. Based on

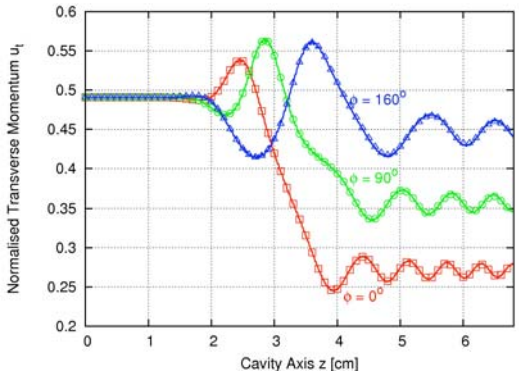

Fig. 1. Evolution of the normalised transverse momentum $u_\perp$ along the cavity with length 68 mm for three electrons differing in initial phase φ_0. A comparison between EURIDICE (points) and Ariadne++ (lines) results is also shown.

TABLE I
Influence of assumptions (i) and (iv) on efficiency

Assumptions in *Ariadne*++ and calculated efficiency η_{elec} (%)			Difference (%) from the efficiency η_{elec} = 35.28 % obtained by *EURIDICE* with assumptions (i), (ii), (iii), (iv)
1.	(i), (iv)	35.13	– 0.43
2.	(i)	35.23	– 0.14
3.	(iv)	36.38	3.12
4.	none	36.49	3.43

TABLE II
Influence of assumptions (v) and (vi) on efficiency

Assumptions in *EURIDICE* and calculated efficiency η_{elec} (%)			Difference (%) from the efficiency η_{elec} = 35.28 % obtained by *EURIDICE* with assumptions (i), (ii), (iii), (iv)
1.	(i)-(iv)	35.28	0.00
2.	(i)-(iv), (v1)	35.74	1.30
3.	(i)-(iv), (v2)	37.23	5.53
4.	(i)-(iv), (vi)	35.77	1.39
5.	all	36.58	3.68

this case of agreement of the results, we were able to investigate the influence of each of the assumptions (i) and (iv), as shown in the next lines of Table I. Next, we also investigated the influence of the assumptions (v) and (vi) by introducing them to EURIDICE. The results are shown in Table II.

In this example, it seems that the assumptions (i) and (v2) have the larger influence, whereas assumption (iv) has the smallest influence. Future work will include studies with increasing number of electrons together with comparisons in different cases. These will include realistic non-homogeneous magnetostatic field, in order also for *Ariadne*++ to consider the influence of the beam self-field on $\mathbf{E}_0$ more realistically. Finally, for the investigation of the effect of the assumptions on gyrotron simulations to be complete, the influence of the differences in the electron trajectories on the RF-field amplitude and axial profile should also be addressed.

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