ANNEX 23

Equilibrium nonlinearity and synergetic stabilising effects of magnetic field and plasma flow[1]

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INTRODUCTION

Motivation of the present study was a previous paper [1] in which the cat-eyes hydrodynamic equilibrium solution describing a row of identical vortices was extended to magnetohydrodynamic plasmas with incompressible flow. In this study it was found that for flows with Alfvén Mach mumbers, M, on the order of 0.01 the pressure surfaces deviate strongly from magnetic surfaces by forming pressure islands in the location of the cat eyes, unlike the weak respective deviation in linear equilibria. Thus, this strong flow effect should be related to the equilibrium nonlinearity. In addition, a flow parallel to the magnetic field results in appreciable stabilising effects. Here we examine the validity of these conclusions in the case of another nonlinear equilibrium solution which in the framework of hydrodynamics describes a row of counter-rotating vortices [2] by extending this solution to MHD for plasmas with incompressible flow of arbitrary direction.

RESULTS AND DISCUSSION

The extended solution reads

$$u = -2\mathrm{arctanh}\left(\frac{\epsilon\cos(x)}{\cosh(\epsilon y)}\right), \tag{1}$$

where the function $u(x, y)$ labels the magnetic surfaces; (x, y, z) are Cartesian coordinates with z corresponding to the axis of symmetry and (x, y) associated with the poloidal plane. The characteristic lines of (1) are shown in Fig. 1 of [2]. The configuration consists of an infinite series of periodic pairs of vortices having magnetic axes on $(x = k\pi, y = 0)$ with k an integer. Also, it holds $u(x = k\pi + \pi/2) = 0$ and therefore boundary conditions at the points $x = x = k\pi + \pi/2$ can be readily imposed. The velocity of the individual vortices of each pair have opposite direction. The magnetic field and current density lie on the velocity or magnetic surfaces and therefore the vortices can be regarded as magnetic islands with plasma flow. The magnetic surface elongation along y gets shorter as the parameter ϵ increases with $\epsilon = \pm 1$ corresponding to point vortices. Furthermore, we examined the impact of the flow on the pressure and the axial current density. As in the case of cat-eyes, fusion pertinent flows with $M \approx 0.01$ in equilibria with $\beta \approx 0.01$ affect drastically the pressure surfaces by forming pressure islands within the counter rotating vortices. The pressure islands are twisted at an angle of $\pi/2$ with respect to the

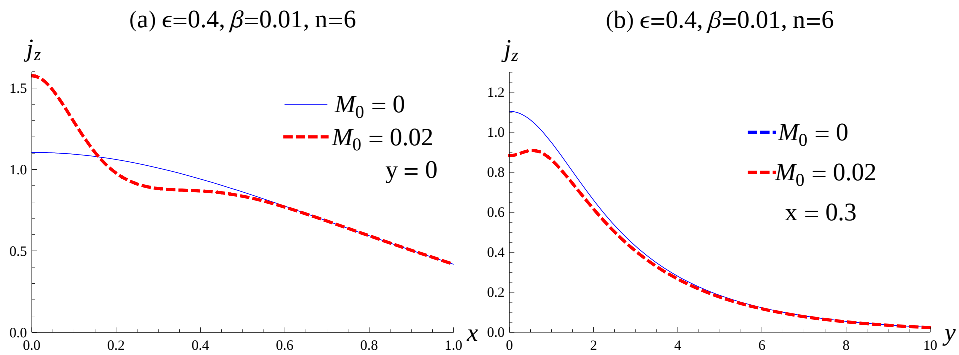

Fig. 1: Profiles of the axial current density. M_0 is the Alfvén Mach number on the magnetic axis and the parameter n relates to the flow shear.

cat-eyes islands. In addition, the axial current density is appreciably modified by the flow irrespective of β (*Fig. 1*). Since such strong flow-caused changes do not occur in linear equilibria the present results confirm the importance of equilibrium nonlinearity.

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The linear stability of the equilibrium is now examined by applying a sufficient condition [3]. This condition states that a general steady state of a plasma of constant density and incompressible flow parallel to the magnetic field is linearly stable to small three-dimensional perturbations if the flow is sub-Alfvénic ($M^2 < 1$) and $A \geq 0$, where A is given by (22) of [3] (with λ therein corresponding to M here). Consequently, we restrict the study to plasmas with parallel flows and constant density. It turns out that a parallel flow and the flow shear in conjunction with the variation of $\mathbf{B}$ perpendicular to the magnetic surfaces have remarkable stabilising effects potentially correlated to the equilibrium nonlinearity. An example showing the sign of A on the poloidal plane is presented in *Fig. 2*. The lighter shaded regions are stable ($A \geq 0$), while in the darker shaded region it holds $A < 0$. The whole area of *Fig. 2* becomes darker when $M_0 = 0$. Note that in the case of a linear sinusoidal solution describing a

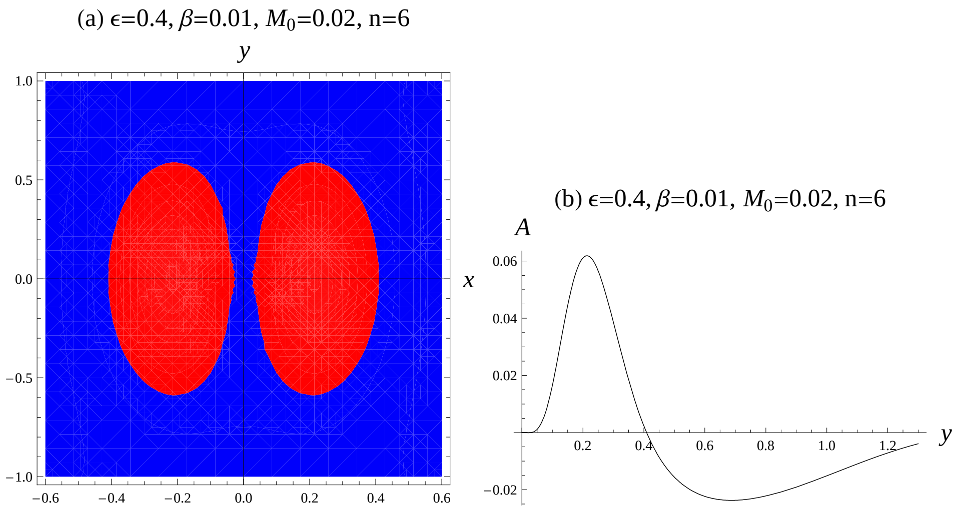

Fig. 2: Stabilisation effect of flow: In the presence of flow the lighter shaded stable regions appear in the diagram (a) where $A \geq 0$. The respective stable window can be seen in the profile of A in (b).

tokamak equilibrium with parallel incompressible flow in toroidal geometry [4], the stability condition is not satisfied for $M \approx 0.01$ because the stabilising flow term becomes three orders of magnitude lower than in the present study. In both the extended cat-eyes and counter-rotating-vortices equilibria, stabilisation is also possible for nested pressure surfaces in connection with large values of β on the order of 0.1; in this respect the flow-caused change of the equilibrium axial current density being independent of β might play a more important role on stability. The stable regions shorten with the magnetic surface elongation along the non periodic direction, broaden with the flow and the flow shear and are rather insensitive to thermal pressure. In addition, a combination of flow and a constant axial magnetic field have synergetic stabilising effects by enlarging the stable region.

FUTURE WORK

The generic validity of the results of the present and cat-eyes studies could be further examined by alternative non-linear solutions. Since in plane geometry the Laplace operator is involved in the equilibrium equation, such solutions can be constructed by using the powerful tool of complex functions. Also, it is interesting to examine the impact of toroidicity by extending the study to toroidal geometry. Because of unavailability of analytic nonlinear solutions in this case, most likely the construction should be performed numerically. Finally, it is noted that in [4] particular low shear Mach-functions were considered. Therefore, to check the importance of the equilibrium nonlinearity on stability, the potential role of the flow shear could be examined in the linear regime by employing Mach functions of the form $M \propto u^n$. More details about the present study are provided in [5].

References

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