ANNEX 24

On the equilibrium and stability of tokamak plasmas with field aligned flow[1]

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INTRODUCTION

In a previous study [1],[2] the quasistatic Herrnegger-Maschke equilibrium solution was extended to plasmas with incompressible flows parallel to the magnetic field. The stability of a particular simple sinusoidal form of the solution constructed was then examined by means of a sufficient condition for linear stability [3]. A conclusion was that unlike to the nonlinear extended "cat-eyes" [4] and counter-rotating-vortices [5] equilibrium solutions the flow does not play a stabilizing role in the sense that the sufficient condition for stability is not satisfied, thus indicating that the equilibrium nonlinearity may activate stabilization by flow. To check this conjecture we extend here the study of [1, 2] to the complete flow-generalized Herrnegger-Maschke solution.

RESULTS AND DISCUSSION

For up-down symmetric equilibria a real extended Hernnegger-Maschke solution can be expressed, alternative to Coulomb wave functions [1], in terms of the Whittaker functions as

$$u(\rho, z) = \alpha \Im \left[\mathcal{M}_{\nu,1/2}(\rho) + \gamma W_{\nu,1/2}(\rho)\right] \cos\left(\lambda z / R_0\right). \tag{1}$$

Here, $(z,\ R,\ \phi)$ are cylindrical coordinates with z associated with the axis of symmetry; $u(R, z)$ the poloidal magnetic flux function; $\mathcal{M}$ and W the Whittaker functions of first and second kind respectively; $\rho = \Im\sqrt{P_{sa}}R^2$, where the parameter P_{sa} is associated with the ansatz for the surface quantity $P_s(u)$ representing the pressure when the flow vanishes:

$$P_s(u) = \frac{P_{sa} u^2}{2}; \tag{2}$$

the parameter ν relates to the magnetic properties of the plasma (either paramagnetic or diamagnetic); $(z = 0, R = R_0)$ is the position of the geometric centre of the configuration; α, γ, λ are real parameters. It may be noted here that (1) was expressed in terms of Whittaker functions because they are build-in functions in *Mathematica* employed for symbolic computation of the quantity A in (4) below. We further considered (1) for a tokamak of rectangular boundary cross-section of height $2b$, width $2a$ and Alfvén Mach functions of the form

$$M(u)^2 = \frac{M_a^2 u^2 \left(X_0^2 + u_a^2 \Lambda_1\right)}{u_a^2 \left(X_0^2 + u^2 \Lambda_1\right)}, \tag{3}$$

where M_a, u_a, X_0 and Λ_1 are constant quantities and the subscript a refers to the magnetic axis. The boundary condition $u(R = R_0 \pm a) = 0$ and $u(z = z_0 \pm b) = 0$ leads to discrete values of λ, γ and ν associated with simply or multiply toroidal configurations [1],[2]. As the flow increases the pressure decreases and can become negative; therefore there are maximum permissible Mach numbers ($|M_a|_{max}$) on the magnetic axis. $|M_a|_{max}$ becomes smaller as the aspect ratio R/a takes lower values and it vanishes for $R/a = 1$; therefore, for a compact toroid the equilibrium should be static.

The aforementioned sufficient condition for linear stability [3] is now applied to the equilibrium described by (1) for singly and multiply toroidal configurations. This condition states that a general steady state of a plasma of constant density and incompressible flow parallel to $\mathbf{B}$ is stable to small three-dimensional perturbations if the flow is sub-Alfvénic ($M^2 < 1$) and $A \geq 0$, where the equilibrium depending quantity A is given by Eq. (20) of Ref. [3]. For an axisymmetric equilibrium A is put in the form

$$\begin{aligned} A \;=\; & -g^2 \Big\{ (\mathbf{J} \times \nabla u)^2 - (\mathbf{J} \times \nabla u) \cdot (\nabla u \cdot \nabla)\, \mathbf{B} \\ & + \frac{1}{2} \frac{dM^2}{du} \left(1 - M^2\right)^{-1/2} |\nabla u|^2 \left[\left(1 - M^2\right)^{-1/2} \nabla u \cdot \frac{\nabla B^2}{2} + g \left(1 - M^2\right)^{-1} |\nabla u|^2 \right] \Big\} \end{aligned} \tag{4}$$

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where

$$g = \left(1 - M^2\right)^{-1/2} \left(\frac{dP_s}{du} - \frac{dM^2}{du}\frac{B^2}{2}\right), \tag{5}$$

with $\mathbf{B}$ the magnetic field and $\mathbf{J}$ the current density. It turns out that the relation ($A \geq 0$) is satisfied in regions where the current density is small enough and the variation of $\mathbf{B}$ perpendicular to the magnetic surfaces large enough so that the second stabilising term of A containing $(\nabla u \cdot \nabla)\mathbf{B}$ overcomes the first destabilising term containing $(\mathbf{J} \times \nabla u)^2$. An example showing the sign of A on the poloidal plane for ITER relevant values of the free parameters is presented in *Fig. 2.* The lighter shaded stable regions close to the boundary may be expected because the equilibrium current density vanishes on the boundary. Also, the stable regions are broader the lower is the aspect ratio R_0/a. Therefore, stabilisation is related to the Shafranov shift which makes the term $(\nabla u \cdot \nabla)\mathbf{B}$ larger in regions on the low field side where the magnetic surfaces are compressed. As in the particular sinusoidal solution [1],[2], the flow does not play a role because the last two flow terms of A are at least three orders of magnitude lower than the first two terms; even in the extreme case of $M_a = (M_a)_{max} = 0.48$, the stability diagram of *Fig. 1* remains nearly unaffected. These results confirm the conjecture that the remarkable stabilising effects of flow in the case of the extended "cat-eyes" and counter-rotating-vortices equilibria [4],[5] relate either to the equilibrium nonlinearity or to the flow shear. For this reason further application of the condition for flows of stronger shear and other nonlinear and linear equilibria is required for possibly better understanding the role of the equilibrium non-linearity, flow shear and toroidicity on stability.

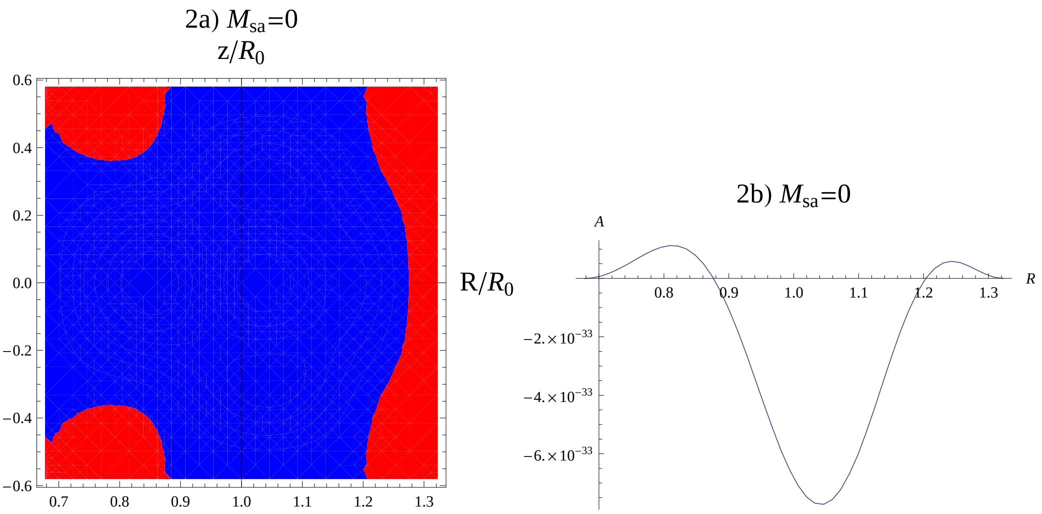

Fig. 1: Stability diagram showing the sign of the quantity A [Eq. (4)] on the poloidal plane normalized with respect to the absolute value at the geometric centre for the static Herrnegger-Maschke equilibrium for ITER relevant parametric values ($a = 2m$, $b = 3.5m$, R_0 =6.2m, vacuum toroidal magnetic field = 5 Tesla, safety factor on axis = 1.1) (Fig. 2a). The lighter shaded regions are stable. The respective stable windows at $z = b$ can be seen in the profile of A in 2b ($M_{sa} \equiv M_a^2$).

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