ANNEX 30

A self-organised criticality model for the reversed field pinch

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INTRODUCTION

In [1], we presented a Self-Organised Criticality (SOC) model for the magnetic field in confined plasmas, with main characteristics that it is formulated in the usual physical variables (the magnetic field components), and that it is physically interpretable in a consistent way. The model is in the form of a cellular automaton (CA) that is compatible with MHD, and in [1] we presented an application of the model to a toroidal confinement device. Here, we make the application more specific to the reversed field pinch (RFP), which is well- known to be a self-organising system, see e.g. [2], in contrast to the tokamak. More-over, we now use polar coordinates in the poloidal plane, since the Cartesian coordinate system used in [1] led to unnaturally looking flux surfaces.

THE MODEL

As in [1], in order to achieve MHD compatibility in the CA model, we use the vector potential **A** as the grid variable, and in order to calculate derivatives, **A** is interpolated, which allows to determine $\mathbf{B} = \nabla\times\mathbf{A}$ and $\mathbf{J} = (c/4\pi)\ \nabla\times\mathbf{B}$ in the usual MHD way, so that e.g. $\nabla\cdot\mathbf{B} = 0$ is ensured. In the Cartesian grid presented in [1], global cubic spline interpolation was used to determine the derivatives, whereas in the polar coordinates r, θ we use here the interpolation is done with global expansion in terms of Fourier polynomials in the periodic θ direction, and with global expansion in terms of Chebyshev polynomials in the r direction.

With intended application the RFP, we choose as an initial condition a relaxed Taylor state in the form of the Bessel function model (see e.g. [2]), where $A_r^0 = 0$ and

$$A_\varphi^0(r) = \frac{B_0}{\mu} J_0\left(\mu r\right), \tag{1a}$$

$$A_\theta^0(r) = \frac{B_0}{\mu} J_1\left(\mu r\right) \tag{1b}$$

(with φ the toroidal angle, B_0 the on-axis magnetic field, μ a constant, and J_0, J_1 Bessel functions of the first kind), which yields simple circular flux surfaces, i.e. a magnetic field $\mathbf{B} = \nabla\times\mathbf{A}$ of the form

$$\mathbf{B}^0(r,\theta) = B_0 J_0\left(\mu r\right)\mathbf{e}_{\hat\varphi} + B_0 J_1\left(\mu r\right)\mathbf{e}_{\hat\theta}\ .$$

For the current of the initial field, $\mathbf{J} \propto \nabla\times\mathbf{B}$, we find J_r^0=0 and

$$J_\varphi^0 \propto -\frac{1}{r}\partial_r\left[r\partial_r A_\varphi\right], \tag{2a}$$

$$J_\theta^0 \propto -\partial_r\left[\frac{1}{r}\partial_r\left(rA_\theta^0\right)\right]. \tag{2b}$$

In the RFP, the poloidal field coils and plasma currents generate the toroidal magnetic field, and the induced (and self-generated) toroidal currents generate the poloidal magnetic field. It thus follows that J_φ and J_θ are driven, which through (2) implies that, in terms of our grid-variables, A_φ and A_θ evolve in a way such that J_φ and J_θ increase. Translating this scenario to CA rules, we systematically increase A_φ and A_θ by adding increments to them,

$$A_\varphi\left(t+1,\mathbf{x}_{ij}\right) = A_\varphi\left(t,\mathbf{x}_{ij}\right) + \delta A_\varphi\left(t,\mathbf{x}_{ij}\right), \tag{3}$$
$$A_\theta\left(t+1,\mathbf{x}_{ij}\right) = A_\theta\left(t,\mathbf{x}_{ij}\right) + \delta A_\theta\left(t,\mathbf{x}_{ij}\right),$$

at one (usually random) grid site i, j at a time, and where the increments are in the direction of the unperturbed fields, $\delta A_\varphi = sA_\varphi^0$, $\delta A_\theta = sA_\theta^0$, with s a constant or a random number. In this way, the two current components increase because either A_φ and A_θ increase or their local curvature increases.

The instabilities are considered to be current driven, and the relaxation processes are of resistive, diffusive nature and are derived from the MHD induction equation, exactly as described in [1], with the only difference that now all three components of the vector potential participate in the diffusive relaxation process.

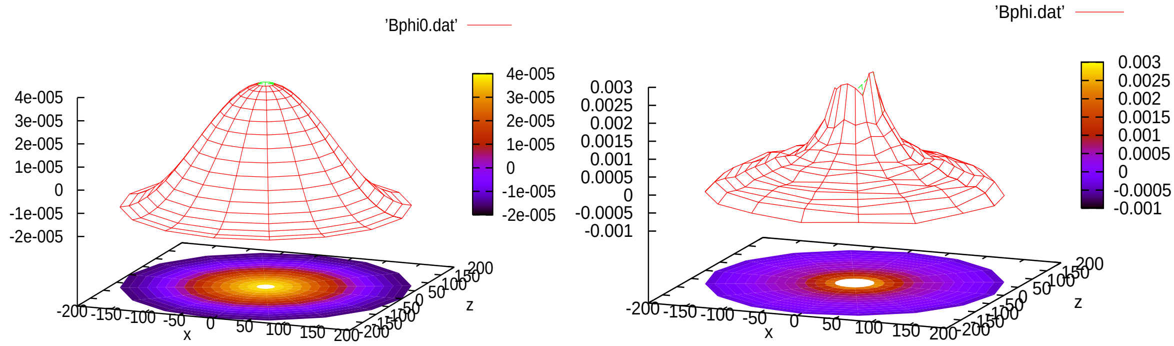

Fig. 1: Initial toroidal field B_φ^0 (Bessel function model; left) and B_φ in the state of SOC (right).

RESULTS

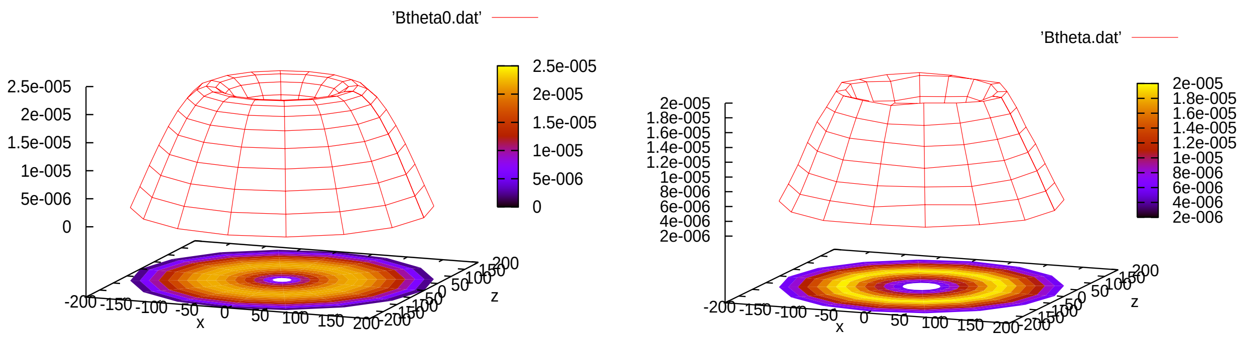

Fig. 2: Initial poloidal field B_θ^0 (Bessel function model; left) and B_θ in the state of SOC (right).

Starting from the initial conditions, the system exhibits a transient phase and then reaches the SOC state as a dynamic equilibrium. In SOC state, the toroidal field B_φ as well as the poloidal field B_θ remain close in shape to the Bessel function model, as shown in *Figs. 1* and *2*. The characteristic shape of the poloidal field includes a change of sign, i.e. a field reversal at the edge. As in the variant of the model in [1], B_φ and B_θ just slightly fluctuate about the characteristic shapes in SOC state, they thus exhibit a very high degree of stiffness. Last, we note the approximate cylindrical symmetry of the fields, due to the polar coordinate system used, in contrast to the respective magnetic fields shown in [1].

CONCLUSION

The SOC model introduced in [1] was adjusted to the case of the RFP, and the Cartesian coordinates were replaced by polar coordinates. The system again reaches the SOC state, in which the magnetic topologies form a dynamic equilibrium and stay close in shape to the Bessel function model.

REFERENCES

[1] H. Isliker, L. Vlahos, "An MHD compatible model for self-organised criticality in toroidally confined plasma", *Annex 32* in *Fusion Project*, *Association EURATOM-Hellenic Republic*, *Progress Report 2008*.

[2] J.B. Taylor, Phys. Rev. Lett. **33**, 1139, (1974); Rev. Mod. Phys. **58**, 741 (1986).