ANNEX 34

Vacuum gas flows through channels with cross sections varying in the flow direction

S. Misdanitis and D. Valougeorgis
Department of Mechanical Engineering, University of Thessaly

INTRODUCTION

Over the last few years the flow of a gas through long channels of various cross sections, under low, medium and high vacuum conditions has been intensively investigated on the basis of kinetic theory [1]-[3]. In all cases the cross section remains constant along the flow direction. However, in fusion related vacuum applications, including the divertor vacuum system, the cross section may vary along the flow. In this context, based on a recent work [4] an “in house” code is developed to solve vacuum gas dynamics flows through channels with cross sections that vary in the flow direction. The flow along the channel may vary from the free molecular, through the transition up to the viscous regime. Since a mesoscale approach is implemented the results are valid in the whole range of Knudsen number.

FORMULATION AND ALGORITHM

Consider the isothermal pressure driven flow of a rarefied gas through a long channel of length L and hydraulic diameter D_h, connecting two reservoirs maintained at pressures P_1 and P_2 respectively, with $P_1 > P_2$. The cross section of the channel may be arbitrary and vary in the flow direction $0 \le z' \le L$, and therefore $D_h = D_h(z')$, while D_{h1} and D_{h2} denote the hydraulic diameters at the inlet and outlet of the channel. The area and the perimeter of the cross section are denoted by $A'(z')$ and $\Gamma'(z')$ respectively, while the reference pressure is defined as $P_0 = (P_1 + P_2)/2$. By taking $D_h(z') << L$, the flow is considered as fully developed and end effects at the inlet and the outlet of the channel may be ignored. The basic parameter of the flow is the rarefaction parameter defined as $\delta(z') = D_h P / \mu_0 \upsilon_0$, where $D_{h1} \le D_h(z') \le D_{h2}$, $P_1 \le P(z') \le P_2$ is the pressure along the channel, μ_0 is the gas viscosity at reference temperature T_0 and $\upsilon_0 = \sqrt{2RT_0}$, with R denoting the gas constant, is the most probable molecular velocity. The rarefaction parameter δ is inversely proportional to the Knudsen number.

The kinetic simulation of this problem is capable of providing the dimensionless flow rate G_P at each cross section of the channel as a function of the local parameter δ [1]. Then starting from the definition of the mass flow rate according to

$$\dot{M} = \int_{A'} \rho(z')\, u(r')dA' \tag{1}$$

and using the equation of state $P = nkT_0 = \frac{1}{2}\rho\upsilon_0^2$ it is readily deduced that the mass flow rate can be expressed in terms of the local dimensionless flow rate G_P as

$$\dot{M} = G_P \frac{A' D_h}{\upsilon_0} \frac{dP}{dz'}. \tag{2}$$

However, the mass flow rate may also be written in terms of reference quantities as

$$\dot{M} = G \frac{A_1' D_{h1}}{\upsilon_0} \frac{P_1}{L}, \tag{3}$$

where the quantity G is a free parameter to be adjusted such that the mass conservation principal along the channel will be fulfilled. By equating the last two expressions for $\dot{M}$ it is deduced that

$$\frac{dP}{dz'} = \frac{P_1}{L} \frac{G}{G_p} \frac{D_{h1}}{D_h(z')} \frac{A_1}{A(z')} \tag{4}$$

Equation (1) is treated as a typical O.D.E. with boundary conditions $P(0) = P_1$ and $P(L) = P_2$ and it is solved for the unknown pressure distribution along the channel by applying a marching numerical integration scheme. The free parameter G is adjusted such that the boundary condition at $z' = L$ is satisfied. At each step the flow rate G_p is obtained from a data bank which has been created based on the kinetic solution. At the end of the integration path the computed $P(L)$ is compared to P_2 and if the corresponding convergence criterion is satisfied then the algorithm is terminated. Otherwise the previous estimation of G is updated and whole process is repeated upon convergence. Through this procedure, in addition to the pressure distribution, a correct estimation of G is obtained. Finally using (3) the mass flow rate is computed.

RESULTS AND DISCUSSION

Based on the former analysis and algorithm, some typical results for demonstration purposes are provided. Helium flow at reference gas temperature of 300 K flowing through a conical tube with length $L = 5$ m, inlet diameter $D_1 = 1$ m and various ratios D_1 / D_2 is considered, while the tube diameter is increased linearly in the flow direction (i.e. $D(z') = D_1 + (D_2 - D_1) z'/L$). The pressure ratio P_2 / P_1 is taken equal to 0.1 and 0.9. Based on the above the reference rarefaction parameter δ_1 takes values in the whole range of rarefaction from zero (free molecular limit) up to 100 (viscous flow).

In *Fig. 1* the pressure distributions for a circular and a conical tube are presented. Comparing these plots it is clearly seen that the pressure drops along the channel are qualitatively different. In the case of a tube with constant cross-section the typical linear pressure drop at high rarefaction, which becomes gradually slightly nonlinear as the rarefaction is reduced, is observed [1]. In the case of a conical tube the pressure distribution, independently of the rarefaction degree, drops rapidly in the first part of the tube and then asymptotically approaches its downstream value. This behaviour is typical for any pressure ratio. In *Fig. 2* the variation of the rarefaction parameter in the case of a conical tube with $D_{h2}/D_{h1} = 10$ is shown for two pressure ratios. When the pressure ratio is 0.9, i.e. the pressure difference between the inlet and outlet of the channel is small, the rarefaction parameter $\delta(z')$ is monotonically increased. This is due to the increased diameter of the tube. However, in the case of large pressure differences ($P_2/P_1 = 0.1$) the variation of $\delta(z')$ is non-monotonic. This is due to a combined effect between diameter increase and pressure drop.

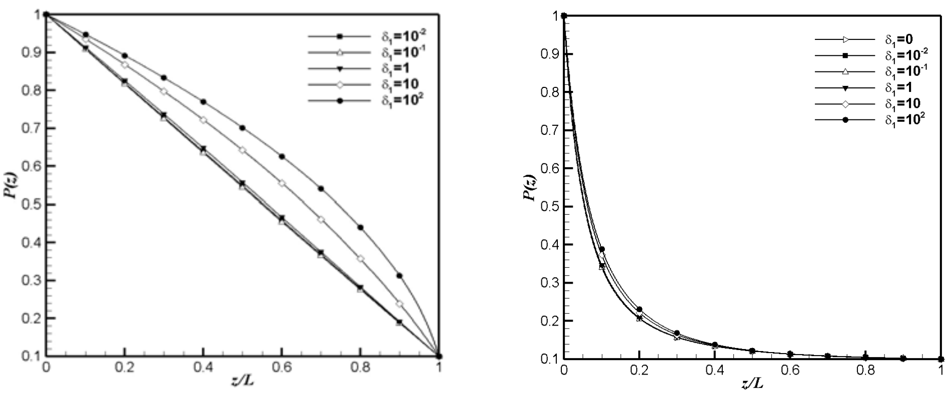

Fig. 1: Pressure distribution $P(z')$ *along a channel with* $D_{h2}/D_{h1} = 1$ *(left) and* $D_{h2}/D_{h1} = 10$ *(right) for* $P_2/P_1 = 0.1$.

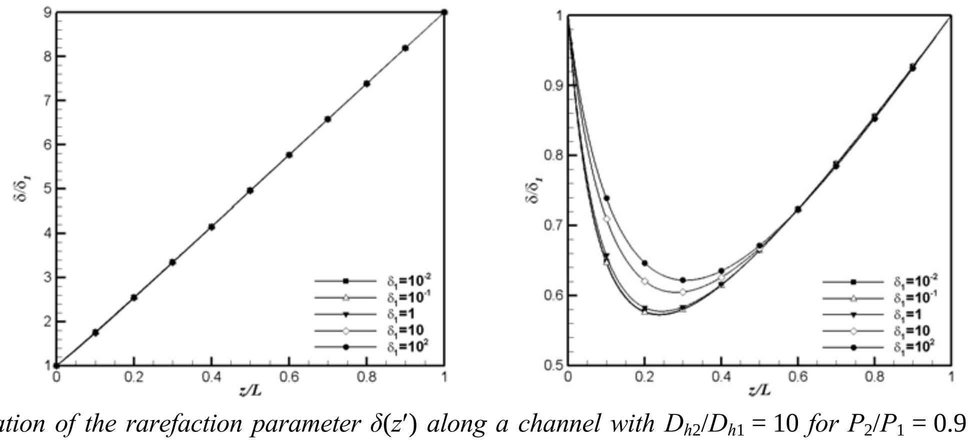

Fig. 2: Variation of the rarefaction parameter $\delta(z')$ *along a channel with* $D_{h2}/D_{h1} = 10$ *for* $P_2/P_1 = 0.9$ *(left) and* $P_2/P_1 = 0.1$ *(right).*

Finally some typical results for the mass flow rate are presented in Table I for flow into vacuum through a conical channel of various outlet to inlet diameter ratios. It is seen that the mass flow rate is always increased as the ratio is increased. It is important to note that the "in house" developed algorithm is applicable in a straightforward manner to any channel with not constant cross section.

TABLE I

Mass flow rate M versus δ_1 through a conical tube at various diameter ratios with $P_2/P_1 = 0$

	$\dot{M}$			
δ_1	$D_2 / D_1 = 1$	2	5	10
0.0	0.15038E+01	0.40101E+01	0.12531E+02	0.27341E+02
0.01	0.14910E+01	0.39703E+01	0.12396E+02	0.27040E+02
0.1	0.14405E+01	0.38212E+01	0.11903E+02	0.25949E+02
1	0.14195E+01	0.38010E+01	0.11906E+02	0.25986E+02
10	0.23891E+01	0.71099E+01	0.23642E+02	0.52290E+02
100	0.13539E+02	0.45380E+02	0.15885E+03	0.35451E+03

REFERENCES

[1] S. Varoutis, V. Hauer, C. Day, and D. Valougeorgis. "Experimental and computational study of gas flows through long channels of various cross sections in the whole range of the Knudsen number". J. Vacuum Sci. & Tech. A, **27** (1), 89-100, (2009).

[2] S. Varoutis, D. Valougeorgis, and F. Sharipov, "Gas flow through tubes of finite length over the whole range of rarefaction for various pressure drop ratios", J. Vac. Sci. & Tech. A, **27** (6), 1377-1391, (2009).

[3] S. Misdanitis and D. Valougeorgis, "Compressible Couette flow far from local equilibrium", paper SE264, *2nd South-East European Conference on Computational Mechanics,* Rhodes, Greece, (2009).

[4] F. Sharipov and G. Bertoldo, "Rarefied gsa flow through a long tube of variable radius", J. Vac. Sci. & Tech. A, **23** (3), 531-533, (2005).