ANNEX 35

Estimation of the exact hydraulic diameter in vacuum gas flows through channels of noncircular cross sections

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INTRODUCTION

The concept of the hydraulic diameter is a fundamental one in order to compute all important quantities such as flow rates, friction factor, etc. for flows through non-circular ducts. According to this approach the flow in a non-circular channel may be approximated by performing the corresponding work in a circular channel having a diameter, which is defined as $D_h = 4A'/\Gamma'$, where A' and Γ' are the area and the perimeter of the non-circular channel and it is known as the hydraulic diameter. Since the computational effort is significantly reduced and unified in the case of circular channels compared to channels of various cross sections, the hydraulic diameter concept has been extensively used. Of course, it is known that this is only an approximation since the mean velocity of the non-circular duct will not be, in general, equal to the corresponding quantity of the circular tube with diameter D_h. Following a specific procedure, the exact hydraulic diameter D_h^{exact} for which the above argument is true may be specified. The procedure is straightforward and at the hydrodynamic limit ($Kn \to 0$), the departure between the exact and the approximate hydraulic diameters have been reported for fully developed flows through ducts of various cross sections.

It is obvious that this approach is very important also in the case of vacuum gas dynamics. However, no relative work has been performed until recently in order to investigate the accuracy of the hydraulic diameter concept in terms of the Knudsen number. Such an investigation has particular interest in the vacuum gas systems of DT fusion reactors since due to certain engineering restrictions their piping networks are consisting of pipe elements of various cross sections.

EXPRESSION FOR THE EXACT HYDRAULIC DIAMETER

The Poiseuille number in most references is defined as

$$Po = \frac{8\overline{\tau}'_w D_h}{\mu_0 \overline{u}'} \tag{1}$$

where D_h is the hydraulic diameter defined above, while $\overline{u}'$ and $\overline{\tau}'_w$ denote the mean bulk velocity and wall shear stress respectively. In a recent work it has been shown that the Poiseuille number in terms of kinetic quantities can be written as [1]

$$Po = 4\delta / G_P . \tag{2}$$

where δ is the rarefaction parameter (proportional to the inverse Knudsen number) and G_P is the corresponding dimensionless flow rate. This expression is valid for any cross section and gas rarefaction level.

To derive an expression for the exact hydraulic diameter, (1) is rewritten in the form

$$Po_{tube} = \frac{8\overline{\tau}'_w D_h^{exact}}{\mu_0 \overline{u}'} , \tag{3}$$

where $\overline{u}'$ and $\overline{\tau}'_w$ refer to the non-circular duct, while Po_{tube} is known and it is associated to a circular channel having approximate (or geometric) hydraulic diameter D_h. Thus, to estimate D_h^{exact}, equation (3) is solved for the exact hydraulic diameter to obtain

$$D_h^{exact} = Po_{tube} \frac{\mu \overline{u}'}{8\overline{\tau}'_w} = Po_{tube} \frac{\overline{u} D_h}{16\delta \overline{\tau}_w} \tag{4}$$

where $\overline{u}$ and $\overline{\tau}_w$ are the dimensionless mean bulk velocity and wall shear stress. Using (3) for the Poiseuille number, the above expression is rewritten in the form

$$\frac{D_h^{exact}}{D_h} = \frac{1}{8\overline{\tau}_w} \frac{Po_{tube}}{Po} . \tag{5}$$

Finally, the fact that the dimensionless mean wall shear stress can be written as $\overline{\tau}_w = D_h^{exact} / \left(8D_h\right)$ results to [2]

$$\frac{D_h^{exact}}{D_h} = \sqrt{\frac{Po_{tube}}{Po}} \,. \tag{6}$$

Expressions (3) and (6) are simple, elegant and valid in the whole range of δ for channels of any cross section. They may be applied to estimate the Poiseuille number of the flow and to study the error which is introduced when the hydraulic diameter concept is used to approximate flows through non-circular ducts.

RESULTS AND DISCUSSION

Some indicative results are resented in Table I for the case of orthogonal channels with four aspect ratios in the whole range of gas rarefaction. It is seen that depending upon the specific case the ratio may be less or larger than unity. The introduced error remains constant and it is relatively small for $\gamma = 1$ and $\gamma = 0.5$, while for $\gamma = 0.1$ and 0.01 remains small in the transition regime and it is significantly increased in the free molecular and slip regimes. The results at $\delta \to \infty$ are in excellent agreement with the ones obtained in the viscous regime using the classical Navier-Stokes approach. It may be concluded that in most cases the hydraulic diameter concept may be applied in vacuum gas dynamics providing accurate results for engineering purposes.

Closing this section it is noted that the hydraulic diameter concept, is more valuable in the case of rarefied (non-equilibrium) flows compared to the case of viscous (equilibrium) flows, since in the former case the required computational effort and complexity to obtain reliable results is significantly increased and therefore it is more tractable to use the hydraulic diameter concept in technological applications.

TABLE I

Ratio of the exact hydraulic diameter over the approximate one in terms of the rarefaction parameter δ for orthogonal channels with various aspect ratios.

D_h^{exact} / D_h				
δ	Orthogonal $\left(\gamma = H / W\right)$			
	$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.1$	$\gamma = 0.01$
0.001	1.05	1.07	1.20	1.46
0.01	1.05	1.07	1.19	1.41
0.1	1.05	1.07	1.16	1.25
0.3	1.05	1.06	1.12	1.16
0.5	1.05	1.06	1.10	1.12
1	1.05	1.06	1.07	1.07
3	1.05	1.05	1.02	1.00
5	1.05	1.04	0.99	0.97
7	1.06	1.04	0.97	0.95
10	1.06	1.04	0.95	0.92
20	1.06	1.03	0.92	0.89
30	1.06	1.02	0.91	0.87
50	1.06	1.02	0.89	0.85
100	1.06	1.02	0.88	0.84
.	.	.	.	.
.	.	.	.	.
∞	1.06	1.01	0.87	0.82

REFERENCES

[1] D. Valougeorgis, "The friction factor of a rarefied gas flow in a circular tube", Phys. Fluids, **19** (9), 091701.1-091701.4, (2007).

[2] G. Breyiannis, S. Varoutis, and D. Valougeorgis, "Rarefied gas flow in concentric annular tube: Estimation of the Poiseuille number and the exact hydraulic diameter", Eur. J. Mech. B/Fluids, **27**, 609-622, (2008).