ANNEX 1

Interaction of Ions with Beating Electromagnetic Waves

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INTRODUCTION AND MATHEMATICAL FORMULATION

The evolution of ion distribution functions in the presence of two beating X-mode electromagnetic wave is treated both numerically and analytically. When the amplitude of the wave is small, the nonlinear interaction with the envelope of the modulated wave can result in a coherent energization of the ions. For larger amplitudes, chaotic energy transport takes place. The purpose of this work is to investigate the values of the parameters of the modulated wave that optimize the energy transfer from the wave to an ensemble of ions as well as to determine the amplitude threshold for chaotic motion and the extent of the chaotic orbits on the phase space.

Consider a single ion moving in a uniform magnetic field interacting with two elliptically polarized plain waves that propagate perpendicularly to the magnetic field. The single particle Hamiltonian is given by

$$\mathrm{h}_0 = \frac{1}{2}\left(\left[p_x + \varepsilon\left(\frac{1}{\nu_1}\cos(\phi_1) + \frac{1}{\nu_2}\cos(\phi_2)\right)\right]^2 + \left[p_y - x + \alpha\varepsilon\left(\frac{1}{\nu_1}\sin(\phi_1) + \frac{1}{\nu_2}\sin(\phi_2)\right)\right]^2\right), \quad (1)$$

where $\nu_i = \omega_i / \Omega_C$ are the normalized wave-frequencies with respect to the ion cyclotron frequency, $\phi_{1,2} = k_{1,2}x - \nu_{1,2}t$ and $\alpha = iE_y / E_x$.

Treating the wave amplitude ε as a small parameter, we apply a canonical perturbation scheme, in order to approximately calculate the evolution of the distribution function of an ensemble of non-interacting ions. We focus our attention to the case where $\Delta\nu_{1,2} = 1 + \delta$, i.e. where the beat frequency is almost in resonance with the unperturbed gyromotion, except for a small detuning $\delta \approx 1$. Moreover $\Delta k \approx k_0$ and $\Delta\nu \approx \nu_0$, i.e. the envelope $F(x,t)$ of the wave varies much more slowly than the carrier wave.

COHERENT MOTION

Making use of a Lie transform perturbation theory, we can separate the first order fast oscillation of the particles, which is due to each wave separately from a slow motion of the oscillation centre (OC) around which the fast oscillations take place. The OC motion can be described by an OC Hamiltonian of the form

$$K = -\delta\mu + \varepsilon^2\left[K_{2,0}(\mu,\delta) + K_{2,1}(\mu,\delta)\exp\left[i\overline{\psi}\right]\right] + cc., \quad (2)$$

where $(\mu, \overline{\psi})$ are the action-angle variables of the unperturbed Hamiltonian, the action being equal to the normalized perpendicular kinetic energy of the particle. The OC Hamiltonian, being time independent is an approximate constant of motion and can be used to determine the topology of the phase space. A typical case is depicted in *Fig. 1*. The structure of the lower part of the phase space is of particular interest, since this is where the bulk of a thermal distribution of particles is located, particles with small Larmor radii $\rho \equiv \sqrt{2\mu}$, are coherently energized, due to the presence of the elliptic point L up to a maximum value determined by the separatrix S, which is typically located at around $\rho \approx \nu_0 - 1$.

The evolution of a distribution function can also be computed by the use of an OC time evolution mapping. Only the secular terms that are in resonance with the unperturbed motion need to be applied iteratively, whereas the non-secular terms can be commuted to the left up to second order in the perturbation parameter and applied only once. The result is an efficient mapping that takes advantage of the timescale separation between the fast oscillations and the slow OC motion. *Fig. 2* depicts the application of the mapping for the calculation of the evolution of the average energy of an ensemble of particles under the influence of a beating cold IC X wave with carrier frequency in the range $5.123 - 18.47\Omega_C$ and $\delta = 8\cdot10^{-6}$.

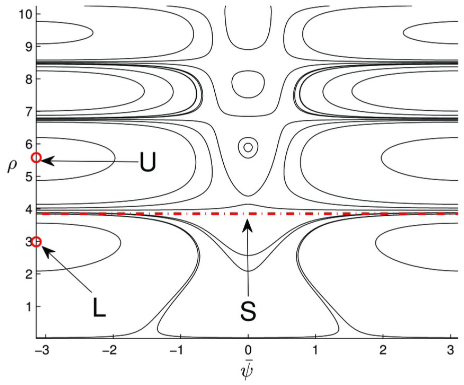

Fig. 1: Contour plot of the OC Hamiltonian for $a = 0.8,\ v_0 = 5.123,\ \Delta k = 0.0861,\ \delta{=}0.$ *The separatrix S acts as a barrier for the energization of the particles.*

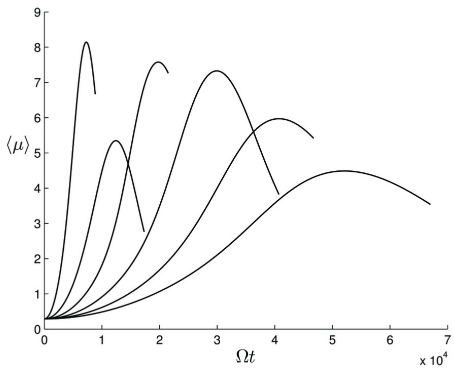

Fig. 2: Average energy vs time for 7 different X mode IC waves with carrier frequency $5.123 - 18.47\Omega_C$ *and* $\delta = 8 \cdot 10^{-6}$. *The time it takes for the energy to reach the maximum value increases with increasing carrier frequency.*

CHAOTIC MOTION

When the amplitude of the waves increases above a certain threshold the separation between the fast oscillation and the OC motion is no longer valid and the motion becomes chaotic. It is then possible for the particles to cross the separatrix S and be chaotically energized to higher energies than what is allowed in the coherent motion regime. This takes place when the ratio of the amplitude of the fast oscillations to the spacing between the two consecutive elliptic points L and U (see *Fig. 1*) exceeds a critical value, which has been calculated numerically. With this criterion we can estimate the minimum wave amplitude required for chaotic motion. The results are depicted in *Fig. 3.* The same criterion enables us to estimate the power law that governs the relation of the upper boundary of the chaotic sea to the perturbation amplitude. For electrostatic waves we have $\rho_{\max} \approx \varepsilon^{2/3}$, while for purely transverse waves we have $\rho_{\max} \approx \varepsilon^{2}$.