ANNEX 6

Scattering by a plasma cylinder in a plasma - formalism

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INTRODUCTION

The problem of scattering by a plasma cylinder in a plasma is treated analytically. The unknown field is expanded in a Fourier integral of plane waves. The field components of these waves are found to be related through a simple algebraic formula. Next we find a similar relation between the expansion coefficients of the unknown field in terms of the cylindrical eigenvectors. The fulfilment of the boundary conditions at the surface of the cylinder is then straightforward.

EXPANSION OF THE UNKNOWN FIELD IN PLANE WAVES

We consider a plasma medium characterized by a dielectric permittivity [1,2]

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}$$

and a scalar magnetic permeability $\mu = \mu_0$. The electric field fulfils the vector wave equation

$$\nabla \times \nabla \mathbf{E}(\mathbf{r}) - k_0^2 \boldsymbol{\varepsilon} \cdot \mathbf{E}(\mathbf{r}) = 0$$

with $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ (the free space wavenumber), and is expressed as a Fourier integral

$$\mathbf{E}(\mathbf{r}) = \iiint d\mathbf{k} \exp(-i\mathbf{k}\cdot\mathbf{r})\mathbf{e}(\mathbf{k})$$

It is found that in order to have a nontrivial solution the wavenumber must fulfil a biquadratic equation

$$\varepsilon_1 k_\rho^4 + [(k_z^2 - k_0^2\varepsilon_1)(\varepsilon_1+\varepsilon_3) + k_0^2\varepsilon_2^2]k_\rho^2 + [(k_z^2 - k_0^2\varepsilon_1) + k_0^4\varepsilon_2^2]\varepsilon_3 = 0$$

where $k^2 = k_z^2 + k_\rho^2$, while the vector components of the Fourier transform must be related, as follows

$$\begin{matrix} e_x(k_z,k_\rho,\varphi_k) \\ e_y(k_z,k_\rho,\varphi_k) \end{matrix} = \left[A(k_z,k_\rho) \begin{matrix} \cos\varphi_k \\ \sin\varphi_k \end{matrix} \mp B(k_z,k_\rho) \begin{matrix} \sin\varphi_k \\ \cos\varphi_k \end{matrix} \right] e_z(k_z,k_\rho,\varphi_k)$$

with

$$A(k_z,k_\rho) = \frac{k_z k_\rho \left(k_z^2 + k_\rho^2 - k_0^2\varepsilon_1\right)}{p\left(k_z,k_\rho\right)},\; B(k_z,k_\rho) = \frac{ik_z k_\rho k_0^2 \varepsilon_2}{p\left(k_z,k_\rho\right)},\; p\left(k_z,k_\rho\right) = (k_z^2 - k_0^2\varepsilon_1)(k^2 - k_0^2\varepsilon_1) - k_0^4\varepsilon_2^2$$

INTRODUCTION OF THE VECTOR WAVE FUNCTIONS

In this plasma medium we introduce a plasma cylinder similar in nature with the ambient plasma but with different parameters say ε_1^C, ε_2^C, ε_3^C. A plane wave is propagating in the ambient plasma and is scattered by the plasma cylinder. To fulfil the boundary conditions at the surface of the cylinder, it is necessary to introduce the cylindrical eigenvectors wave functions $\mathbf{M}$, $\mathbf{N}$, $\mathbf{L}$. So we have to expand a vector plane wave $\mathbf{F}\exp(-i\mathbf{k}\cdot\mathbf{r})$ in terms of these cylindrical functions. Following the general procedure outlined in the literature we end up with the expression [2,3]

$$\mathbf{F}(k_z,\varphi_k)\exp(-i\mathbf{k}\cdot\mathbf{r}) = \sum_{m=-\infty}^{\infty} i^{-m}\exp(im\varphi_k)\left[A_M \mathbf{M}_m(\mathbf{r},\mathbf{k}) + A_N \mathbf{N}_m(\mathbf{r},\mathbf{k}) + A_L \mathbf{L}_m(\mathbf{r},\mathbf{k})\right]$$

where the definitions of the vector wave functions have been given in [3], while the various expansion coefficients are given by

$$A_M = 2k_z k_0^2 \varepsilon_2 / p(k_z,k_\rho),\; A_N = \frac{2}{k}\left\{1 - \frac{k_z^2(k^2 - k_0^2\varepsilon_1)}{p(k_z,k_\rho)}\right\},\; A_L = \frac{2ik_z}{k^2}\left\{1 + \frac{k_\rho^2(k^2 - k_0^2\varepsilon_1)}{p(k_z,k_\rho)}\right\}$$

So introducing a Fourier expansion for the e_z , of the form

$$e_z = \sum_{n=-\infty}^{\infty} a_n \exp(-in\varphi_k)$$

the final expression for the electric field in a plasma medium is

$$\mathbf{E}(\mathbf{r},\mathbf{k}) = \pi \sum_{\mathbf{q}=\mathbf{1}}^{\mathbf{2}} \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} i^{-m} a_{qm} \left[A_M \mathbf{M}_m(\mathbf{r},\mathbf{k}) + A_N \mathbf{N}_m(\mathbf{r},\mathbf{k}) + A_L \mathbf{L}_m(\mathbf{r},\mathbf{k}) \right]$$

Here the summation extends over both modes of propagation for the scattered wave, as well as for the induced wave in the interior of the plasma cylinder, while for the incident wave it may contain one or both of the modes [4]. Next the implementation of the boundary conditions comes, that is the continuation of the φ and z components of electric and magnetic field at the surface of the plasma cylinder. This procedure is under consideration.

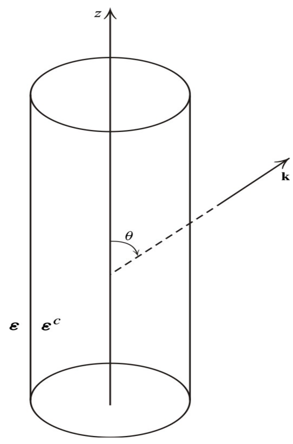

Fig. 1: Plasma cylinder.

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