ANNEX 19

3D Stability analysis of MHD flows

D. Dimopoulos and N. Pelekasis
Department of Mechanical Engineering, University of Thessaly, Volos 38334, Greece

INTRODUCTION

Experimental observations of liquid metal flows in rectangular cavities, in the presence of strong magnetic field effects, indicate the appearance of two-dimensional rolls that are aligned with the magnetic field. However they lose stability to a travelling wave instability that also affects the heat removal capabilities of the cavity. Linear stability analysis that was performed for this type of flow arrangement indicates the onset of transient convection as a result of hydrodynamic interaction of the emerging standing vortices once their strength exceeds a critical value [1]. Nonlinear numerical simulations are performed as a means to investigate the nonlinear evolution of transient convection obtained by linear analysis. The numerical methodology that is used entails coupling of the finite element method for the discretization of the cavity cross section with a spectral approach for discretizing the z-direction. As Ha increases discretization of the emerging Hartmann and side layers formed near the vertical and horizontal walls of the cavity is a challenging task and require an accurate discretization of the cross section. Then as transitional conditions are approached the number of Fourier modes required for capturing nonlinear evolution of the unstable eigenmodes increases and so does the computational load. A semi/implicit time integration scheme is used in order to afford mode decomposition and parallelization. An extensive numerical investigation is conducted in the Hartmann-Grashof parameter space, in order to investigate the dynamics and identify possible roots to turbulence, including the onset of quasi-2D structures. Furthermore the possibility of employing the particular dynamic behaviour for optimizing heat transfer in the rectangular cavities comprising the blanket modules of ITER, and for simulating nonlinear MHD flow in toroidal geometries with resistive layers in the poloidal plane, will be explored.

FORMULATION AND NUMERICAL SOLUTION

Nonlinear analysis is carried out of the flow arrangement in [2], seeking a description of the nonlinear evolution of unstable modes beyond criticality, coupling finite element methodology with a spectral approach. A semi-implicit time integration scheme is employed, with the nonlinear convective terms treated explicitly using second order Adams-Bashforth method and linear terms treated in an implicit manner using second order accurate Crank-Nicolson so that decoupling of the different spectral modes is possible while favouring parallel treatment of the solution process.

This methodology bears significance on the study of nonlinear MHD flows in toroidal geometry where the poloidal plane is discretized with finite elements, while a spectral approach is employed for the toroidal direction. In fact, there is a need to study the full MHD formulation in toroidal tokamak reactors via high accuracy finite element methodology in order to capture plasma instabilities that are radially localized with strong current sheets [3,4]. In addition, the appearance of narrow dissipative layers along the poloidal plane in the full MHD equations renders the finite element methodology the preferred alternative. The above studies employ high accuracy finite elements, e.g Bicubic Bezier surfaces [3], as a means to interpolate unknown functions, coupled with h refinement for mesh generation. Currently, there is an ongoing effort to couple the finite element methodology in the poloidal plane, including the hp mesh refinement techniques, with Fourier spectral modes in the toroidal direction in order to capture the plasma dynamics. The implementation of the nonlinear analysis, that we adopted, is expected to have a direct impact on the above research effort.

Since the most time consuming part of the algorithm is that of the evaluation of the different Fourier modes and involves matrix inversion, different parallelization strategies are employed, i.e. in the physical and in the Fourier space. In order to optimize in terms of storage and CPU time requirements the ScaLAPACK (Scalable Linear Algebra PACKage) library and FFTE (Fast Fourier Transform East) package are employed. The MPI (Message Passing Interface) and BLACS (Basic Linear Algebra Communication Subprograms) protocols are adopted in order to coordinate communication between different computational nodes. Domain decomposition is also performed by every processor corresponding to different regions of the cavity cross section and consequently different parts of the banded system matrix [1]. It is important to note that the transformations between physical and Fourier space are equivalent to a parallel matrix transposition and are achieved using the ALLTOALL command of the MPI library [5].

For the sake of brevity, we present below the equation of momentum along y-axis in the discretized form. The variables with the symbol of ^ correspond to the transformed variables in the Fourier space; the indices k, e, n represent the k-th Fourier mode, element e and z-plane respectively; Φi(x,y), Ψi(x,y) are the biquadratic and bilinear Lagrangian basis functions, respectively.

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+Gr^{-1/2}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+T-\frac{Ha^2}{Gr^{1/2}}\left(v+\frac{\partial \phi}{\partial z}\right)\Rightarrow$$

$$\left[\frac{1}{\Delta t}\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e+\frac{Gr^{-1/2}}{2}\left(\sum_{j=1}^{N}\int_{\Omega_e}\left(\frac{\partial\Phi_i}{\partial x}\frac{\partial\Phi_j}{\partial x}+\frac{\partial\Phi_i}{\partial y}\frac{\partial\Phi_j}{\partial y}\right)d\Omega_e+\left(\frac{2\pi k}{L}\right)^2\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right)\right]\delta\hat{v}_{k,j}$$

$$-\left[\frac{1}{2}\sum_{j=1}^{N}\int_{\Omega_e}\frac{\partial\Phi_i}{\partial y}\Psi_j d\Omega_e\right]\delta\hat{p}_{k,j}-\left[\frac{1}{2}\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\delta\hat{T}_{k,j}+\left[\frac{1}{2}\frac{Ha^2}{Gr^{1/2}}\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\delta\hat{v}_{k,j}$$

$$+\left[\frac{1}{2}\frac{Ha^2}{Gr^{1/2}}\left(\frac{2\pi k}{L}I\right)\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\delta\hat{\phi}_{k,j}=\frac{3}{2}\left(\hat{h}_{yk}\right)_i^n-\frac{1}{2}\left(\hat{h}_{yk}\right)_i^{n-1}$$

$$-\left[Gr^{-1/2}\left(\sum_{j=1}^{N}\int_{\Omega_e}\left(\frac{\partial\Phi_i}{\partial x}\frac{\partial\Phi_j}{\partial x}+\frac{\partial\Phi_i}{\partial y}\frac{\partial\Phi_j}{\partial y}\right)d\Omega_e+\left(\frac{2\pi k}{L}\right)^2\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right)\right]\hat{v}^n_{k,j}$$

$$+\left[\sum_{j=1}^{N}\int_{\Omega_e}\frac{\partial\Phi_i}{\partial y}\Psi_j d\Omega_e\right]\hat{p}^n_{k,j}+\left[\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\hat{T}^n_{k,j}$$

$$-\left[\frac{Ha^2}{Gr^{1/2}}\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\hat{v}^n_{k,j}-\left[\frac{Ha^2}{Gr^{1/2}}\left(\frac{2\pi k}{L}I\right)\sum_{j=1}^{N}\int_{\Omega_e}\Phi_i\Phi_j d\Omega_e\right]\hat{\phi}^n_{k,j}$$

where

$$\left(\hat{h}_{yk}\right)_i=\sum_{n=0}^{N-1}\int_{\Omega_e}\Phi_i\left(u_n\frac{\partial w_n}{\partial x}+v_n\frac{\partial w_n}{\partial y}+w_n\frac{\partial w_n}{\partial z}\right)Exp\left[-\frac{2\pi kn}{N}I\right]d\Omega_e$$

represents the k-th mode of the spectral representation of the convective term.

RESULTS AND FUTURE WORK

The Rayleigh - Benard case is examined, in the presence of a magnetic field with increasing intensity. Linear stability analysis that was developed (see previous Annex for more details concerning the methodology) captures Gr_{Cr}~Ha^2 [6] with great accuracy in comparison with the quasi 2d analysis findings and the experimental ones for the onset of steady convection, (*Fig.1*). We observed that Gr_{Cr} does not depend on c_S but on c_H for large Ha, where c_S, c_H correspond to the electric conductivity ratios between the wall material comprising the side (stainless steel) and Hartmann (copper) walls, respectively. In particular, as c_H is increased then Gr_{Cr} is also increased and this finding is important concerning the stabilization of the liquid metal (NaK) flow and ultimately the optimization of heat transfer in the rectangular cavities. *Fig. 2a, 2b* represent the distribution of the vorticity component parallel to the magnetic field. It is important to note that we took advantage of symmetry along x-axis in order to increase the accuracy of the obtained numerical results. It is obvious that the nature of the eigenvector does not conform with the quasi 2d structure and the side and Hartmann layers are not well defined. Furthermore, the experimental finding of time-dependent convection in [2] cannot be attributed to a thermal instability, (*Fig.1*), because stability analysis on a base state determined by the linear temperature profile, does not support the onset of time-dependent modes.

Therefore, an extensive parametric study is conducted regarding nonlinear stability of liquid metal flow in the presence of a magnetic field, in the parameter space defined by Ha, Gr and the electric conductivity ratio between the cavity walls and the liquid metal NaK. In particular, Ha and Gr values in the range $100 \le Ha \le 1000$ and $2x10^5 \le Gr \le 2x10^7$ will be tested in order to fully map the dynamic behaviour of the system including the details of transition to turbulence. A mesh of 10000 quadratic elements will be used for the discretization of the cross section coupled with the 20-30 most important modes describing variations in the longitudinal direction in the sense that the number of Fourier modes that will be involved in a nonlinear simulation will be 20-30 modes, whereas the number of matrix inversions required at each time step will be increased proportionally to the number of Fourier modes. In addition, mesh generation will be optimized along the cross section based on a-posteriori error estimates, in order to capture the Hartmann and side layers that develop as the intensity of the magnetic field increases..

Implementation of the above project will provide further support to the existing collaborations of our MHD group with the MHD groups of the EURATOM Associations of the Free University of Brussels and the Karlsruhe Institute of Technology.

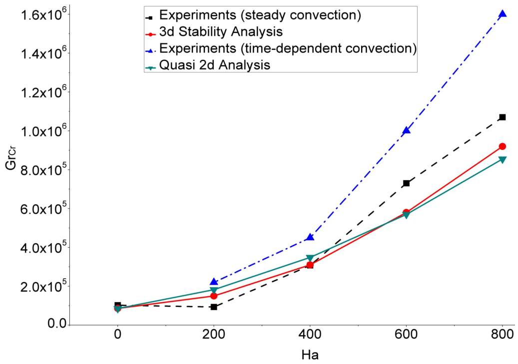

Fig. 1: Neutral stability diagram.

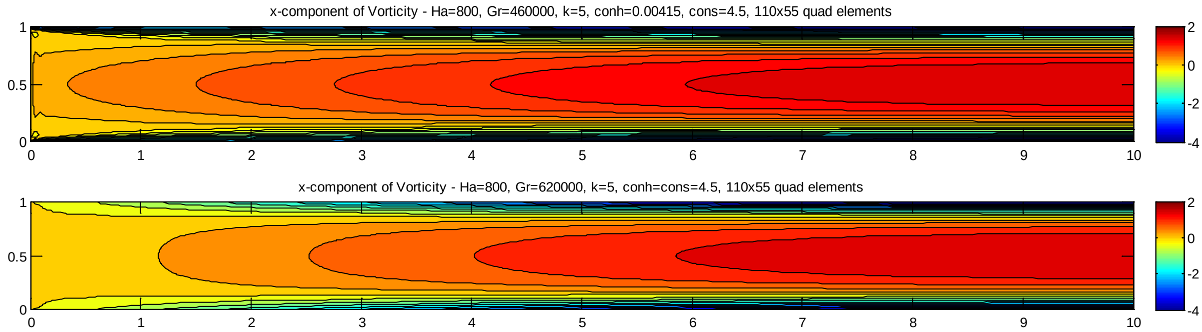

Fig. 2: Symmetric x-component of vorticity; (a) Ha=800, Gr_{Cr} =460000, c_H=0.00415, c_S=4.5 (up) and (b) Ha=800, Gr_{Cr} =620000, c_H=c_S=4.5 (down).

REFERENCES

[1] D. Dimopoulos and N. Pelekasis, "Three dimensional stability of free convection vortices in the presence of a magnetic field", Fluid Dyn. Res. **44**, 031405 (2012).

[2] U. Burr and U. Müller, "Rayleigh-Benard convection in liquid metal layers under the influence of a horizontal magnetic field", J. Fluid Mech **453**, 345 (2002).

[3] O. Czarny and G. Huysmans, "Bezier surfaces and finite elements for MHD simulations", Journal of Computational Physics **227**, 7423 (2008).

[4] G. Huysmans, "Implementation of an iterative solver in the non-linear MHD code JOREK", *Technical Report Project Aster*, ANR-CIS-2006-001-02 (2006).

[5] D. Vanden-Abeele, G. Degrez and D.O. Snyder, "Parallel turbulent flow computations using a hybrid spectral/finite-element method on Beowulf clusters", *ULB Internal Report.*

[6] D. Dimopoulos and N. Pelekasis, "Magnetic field effects on three dimensional stability of natural convection flows in differentially heated cavities", *XXIII ICTAM*, 19-24 August 2012, Beijing, China.