B3 Development of Concept Improvements and Advances in Fundamental Understanding of Fusion Plasmas

3.4 Theory and modelling

3.4-1. Stationary MHD modes in magnetically confined plasmas

Background and Objectives: This activity aims at constructing equilibria and relaxed states of laboratory plasmas of fusion concern (e.g., plasmas of tokamaks and reversed-field-pinches) with flow or/and finite conductivity and investigating their linear and nonlinear stability. Understanding these issues can contribute to improving the current magnetic confinement systems and possibly developing new ones.

Work performed in year 2007 (*in co-operation with IPP*):

1. The nonlinear equation governing the steady states of magnetically confined plasma in plane geometry has been transformed to a simpler analytically solvable form. Exact solutions have been derived representing either a row of identical vortices, which can describe magnetic island configurations or zonal flows. The former is an extension of the Kelvin-Stuart “cat-eyes” solution. Equilibria with counter rotating vortices have also been constructed. On the basis of these solutions, profiles for the equilibrium quantities, i.e. the electric and magnetic fields, the current density, the velocity and the pressure, are systematically examined along with the impact of the flow on the equilibrium characteristics. This last part of the study, however, is not yet complete. The reason relates to a previous paper on the non-existence of tokamak equilibria with purely poloidal flow in the framework of macroscopic theories [Throumoulopoulos, Weitzner, Tasso, Phys. Plasmas **13**, 122501 (2006)]. Since poloidal flows play an important role in the L-H transition, we gave priority to further considering this problem during the year 2007 in the framework of Vlasov theory. This study revealed the importance of toroidicity [Tasso and Throumoulopoulos, J. Phys. A: Math. Theor. 40, F631 (2007)] i.e steady states with purely poloidal flow near axis are possible in cylindrical geometry in accordance with macroscopic theories, while in the presence of toroidicity the problem requiring the complete set of constants of motion becomes very difficult and needs further investigations.
2. We have constructed a Lyapunov functional and then derived a sufficient condition for the linear stability around steady states for three-dimensional internal incompressible perturbations in connection with plasmas of constant density and flows parallel to the magnetic field. The study was based on the energy principle by Ilin and Vladimirov [Phys. Plasmas 11, 3586 (2004)]. However, we noticed that the sufficient condition for linear stability of magnetically confined plasmas with incompressible flows parallel to the magnetic field derived therein and related to this task is incorrect. For this reason we modified the work plan to derive the correct condition. The correct condition is presented in **Ann**

ex XXVI and it involves physically interpretable terms related to the magnetic shear and the flow shear.

3. The impact of inhomogeneous flow parallel to the magnetic field on the phase mixing and dissipation of Alfvén waves was examined in connection with plasma transport reduction and thereby barrier formation. Parallel flow is of particular interest because, unlike the perpendicular flow, does not suffer damping. It is found that owing to the inhomogeneity of parallel flow the phase-mixing takes place even if the plasma is otherwise homogeneous. Also, it is the flow curvature (second spatial derivative of flow) and not the flow shear (first spatial derivative of flow) which adds to the phase mixing and dissipation of the Alfvén wave in consistence with an earlier study on micro-instabilities. Details are presented in **Annex XXVII**.

3.4-2. Turbulence and transport phenomena

Background and Objectives: In this activity, we study diffusion in turbulent plasmas with the use of the Continuous Time Random Walk (CTRW) model, which we extended to comprise the combined diffusion in position and velocity space. The plasma is assumed to be a complex system, with sporadic localised electric fields appearing, with which the particles interact, as well as with regions where the particles may be trapped. The aim is to identify conditions for enhanced or suppressed diffusion. The main benefit of the CTRW approach is that it allows us to model the complex dynamics of particle transport on a global scale, by using statistical laws for the uncountable interactions on small scales. Additionally, this activity concerns the exploration of a particular turbulence model that is based on the concept of Self-Organised Criticality (SOC). The turbulent plasma is considered as a complex system, in which localised instabilities relax small-scale field inhomogeneities. It has been claimed that confined plasmas are in the state of SOC, and our aim is to investigate this claim and to identify the consequences of the SOC state on particle transport with the use of the Extended Cellular Automaton (X-CA) model, a fully MHD compatible SOC model that allows detailed and physically interpretable studies of SOC in turbulent plasmas. Finally, we investigate the use of the mapping technique for modelling the stochastic, turbulent, global structure of the magnetic field. Mapping techniques have so far been used as a successful global model for stochastic field line transport, they do though yield no information on the magnetic field itself. This task includes the formulation of the field evolution in an appropriate Hamiltonian form, and the construction of a mapping with the use of a symplectic integration scheme.

Work performed in year 2007 (*in collaboration with ULB*):

1. Turbulent transport in Tokamak plasmas: We implemented a second choice for the distribution of increments, where they dynamically depend on the local density or temperature gradient (the so-called critical gradient model). Applications to off-axis fuelling have been

made, with the position increments depending on the density gradient, whereby we are able to reproduce published results (for position space only). We though newly found that the dynamics in momentum space have a crucial influence on the confinement and the degree of profile stiffness attained. For both variants, the mixed model and the critical gradient model, respectively, the particle and the energy confinement times were determined, as well as the particle and the heat fluxes, performing a parametric study in the off-axis fuelling case, where the strength of the off-axis source was varied. Finally, the problems in determining heat and particle diffusivities in the non-local CTRW model were studied. (For details see **Annex XXVIII**.)

2. Self-organised criticality models (X-CA model): Work on this activity was started with the implementation of a tokamak-like magnetic topology in the X-CA model. Suitable initial conditions were determined from a simple tokamak equilibrium expressed in analytical form and in terms of the vector potential (simple s – α geometry). The boundary conditions and a loading process were chosen with the aim the topology in SOC state to be tokamak-like. So far, it was shown that the system reaches the SOC state, with the SOC topology of cylindrical symmetry. The radial dependence of the magnetic field magnitude and the safety factor profiles are though not yet physically relevant and the loading process has to be improved. Work on this task was interrupted in order to proceed with the related activity (iii) and will continue in 2008.

3. Stochastic magnetic fields from mapping techniques: As an additional activity, several ways to determine the magnetic field and the current from the tokamap were tried and shown not to be viable, so that a mapping directly for the magnetic field has to be constructed. Starting from the tokamap, a variable transformation from the mapping variables to, as a first example, the safety factor and a new canonical angle was performed. The resulting evolution equations were turned into a mapping with the use of the (large step) symplectic Euler integration scheme. The mapping found in this way is of implicit form and required the numerical solution of two nested non-linear equations. First results showed that the mapping is stable, the stochastisation process inherent in the mapping technique is though more intense than in the tokamap. With work on this activity, we anticipated work planned for 2008, since it is potentially relevant for comparative purposes and as a guiding help for adjusting the X-CA model to a tokamak magnetic topology, and also for the use as a perturbed global background magnetic field in the study of the propagation of EC waves [see 1.5-1. (iii)]. (For details see **Annex XXIX**.)

3.4-3. Discrete kinetic and stochastic models for transport

Background and Objectives: Discrete Kinetic simulations provide a mesoscale description of the transport properties of physical systems implementing kinetic equations. The benefit of this description arises from avoiding expensive computations present in classical micro and macroscopic approaches. The benefits of discrete kinetic simulation are compounded by the inherent local and thus highly parallelised nature of kinetic descriptions. This activity aims to develop computer codes based on suitable kinetic equations capable to simulate, in a computationally efficient manner dissipative MHD flows and fusion related vacuum flows and systems. In the latter case the system can be simulated in a unified manner for any type of vacuum conditions (high, medium, low). Finally, with regard to stochastic modelling, this work concerns the study of anomalous transport, an important issue in fusion plasmas, and it is aimed to develop consistent schemes beyond the quasi-linear approximation.

Work performed in year 2007 (*in co-operation with the Institutes indicated*):

2. MHD flows in complex configurations using lattice (Boltzmann) kinetic schemes (*in co-operation with ENEA*): In an effort to validate the LK3D code for integration within a hybrid scheme (with ENEA's PIC code) we have been working, as a benchmark case, on the evolution of continuous shear Alfvén waves in cylindrically symmetrical plasma. However, this effort has been subjected to several pitfalls and various numerical constraints. For example, the reference test case is in cylindrical geometry while the LK3D code utilises Cartesian coordinates and therefore a consistent reproduction of the initial data as well as a direct comparison of the results is not straightforward. In addition, lattice kinetic schemes are tailored to a MHD dissipative system analysis and not to an ideal MHD stability analysis. It is therefore essential to test and validate and even extend the algorithms range in order to achieve quantitative verification. We have been working towards resolving these issues. Preliminary benchmarks are promising but inconclusive. In particular, the accuracy required demands a highly efficient code and accordingly capable machines to run it. It is believed that in the near future the Gateway machine will provide that. Future development will be considered as the scheme matures and insight provides a clear advantage of LK3D. (*This work is performed in the context of ITM5-T11. Future work is examined as part of the proposed 'GOTiT' project under the "EFDA Goal Oriented Training Programme" initiative*.)

3. Vacuum flows and systems related to fusion reactor applications (*in co-operation with FZK*): All objectives set for 2007 have been successfully completed (see **Annex XXX**). In particular, a detailed comparison between computational and experimental results for flows in long channels of various cross sections (triangular and trapezoidal) under low, medium and high vacuum conditions has been performed. The computational results are obtained by

the kinetic codes of the UoThly team, while the experimental work has been performed at the TRANSFLOW facility of FZK. Since the computational results are based on a rigorous theoretical background (kinetic theory of gases) and they are considered of benchmark quality, it has been possible through the comparison process to improve and validate the experimental procedure and accuracy. In all cases the agreement between the computational and experimental results is excellent.
The validation of the experimental facility is of major importance
. In addition, a
stochastic
algorithm (started in 2006) based on the direct simulation Monte Carlo (DSMC) method, which is very suitable for rarefied gas flows,
has been completed. The flow of a mono-atomic gas through an orifice and a circular tube of finite length have been investigated in detail. It is noted that our well developed kinetic and DSMC solvers, for linear and nonlinear flows, respectively, provide a solid and powerful computational tool for handling complex vacuum flows in the whole range of the Knudsen number.

4. In addition, work that started in the previous years, concerning the Fokker-Planck equation, has been continued in 2007. A procedure is being developed to calculate the time evolution of a particle density distribution function, especially when the latter satisfies a Fokker-Planck type equation. The procedure is based on the generalisation of the Lagrange expansion to functions depending on several variables (positions and velocities). The expression is approximated by a discrete algebraic equation in order to allow numerical calculations, and a suitable computer algorithm is being developed. The
method is being tested on stochastic acceleration models, in the quasi-linear approximation, with various diffusion coefficients. It should be noted that this expansion could also be used to interpolate between an initial and a final distribution, thus, determining the particle displacement vectors that provide the time-evolution.

3.4-4. 2-D MHD code for plasma trapping in fusion devices

Background and Objectives: Numerical simulations on plasma trapping by an open high magnetic field configuration in cylindrical geometry are investigated. The novelty of our work is based on the introduction of the shock tube model [in collaboration with IESL-FORTH], in order to describe correctly the particular initial temporal and spatial conditions of the laser plasma production and the physical process during the plasma expansion resulting from these conditions. In fact the plasma is produced by the interaction of ultrashort high intensity laser beam interacting with a molecular beam of neutral deuterium clusters that enters into the external applied magnetic field configuration. The interaction volume consists of two regions: The first concerns the high density high temperature plasma produced by the pulsed laser and the second the lower density one (at least two orders of magnitude) due to the tails of the laser focal spot. These two regions form a configuration similar to the model of a shock tube, which is formed instantaneously during the ultra-short laser pulse, compared to the plasma expansion velocity of the high temperature plasma. The high density, high temperature region of the plasma expands into the lower density plasma region forming the shock wave. The result is the fast decrease of the plasma density in the high-density plasma region and the decrease of the nuclear collisions. The laser-plasma interaction produce plasma density up to 1019 cm-3 and accelerate D ions to high kinetic energies up to 50 keV enabling the production of neutrons up to 1012 per laser shot and for an external applied magnetic field of the order of 200 Tesla which allows decreasing the expansion velocity and contributing to the improvement of the temporal trapping of the plasma. The objectives are to describe initial plasma conditions for the simulation in order to increase the trapping time, to maintain a plasma in the interaction region with a density as high as possible and for a longer time and to decrease the expansion velocity by applying a relatively moderate value of the external magnetic field. The main requirement is to maintain (i) the laser beam intensity constant in order to use table-top system and (ii) the resulting energy of the produced ions up to 45-50 keV to valorise the (D, D) or (D, T) nuclear reactions.

Work performed in year 2007 (*in co-operation with CEA/Saclay and Ecole Polytechnique Paris*) :

1. The appropriate model has been defined, to describe a plasma volume formed by two separate regions with different plasma densities and different spatial plasma occupation. The first has the dimensions of the laser spot and the second a few times the laser spot. The 1.5-D MHD code in cylindrical coordinates is based on the shock tube model because can describe with a very good precision the initial plasma conditions imposed by the plasma production and the temporal evolution of the physical plasma parameters in both regions. The external magnetic field is in the z-direction and the temporal evolution of the plasma parameters depend essentially on radial (*r*) and longitudinal (*z*) directions. Such a description can be applied for existing experiments and could be used to design potential future experimental set-up in order to improve the neutron production. In the 1.5-D MHD code typical values for the radial dimension of the first region are 50-100 μm and 1-2 mm for the longitudinal, which are relatively small compared to the volume of the applied external magnetic field produced by the mirror like configuration. Typical initial plasma conditions have been selected for the electronic density up to 10^{19} cm^{-3} and for the temperature up to 50 keV, in order to allow comparisons with recent experiments. We investigate the influence of the initial spatial dimension (radial dimension of first region) of the produced plasma on the trapping time of the plasma in the external magnetic field when the initial conditions for plasma density, ρ , and magnetic field

B remains the same. In fact the initial plasma volume in the first region (laser spot) can be modified by increased the diameter of the laser-plasma interaction volume. For a magnetic field *B* = 100T and plasma density $\rho \sim 10^{19}$ cm^{-3} the interaction region of high ionisation (first region) was selected for the different runs of the numerical code equal to 25 μm, 50 μm and 100 μm.

2. The development of a 1.5-D MHD code in cylindrical coordinates was investigated for the study of the spatial and temporal evolution of the trapped plasma in a high magnetic field. Computational results of the spatial and temporal evolution of the physical parameters such as density, pressure, expansion velocity, pressure (gas and magnetic) and temperature, as well as the intensity of the external magnetic field are investigated. The neutron production efficiency is calculated as a function of the physical and geometrical parameters of the magnetized plasma. The numerical results show that as the plasma diameter increases, the plasma expansion is slower and the plasma density is maintained high for much longer time intervals respectively. The plasma density is maintained high due to the trapping process in the external high magnetic field. Changes were observed on the spatial and temporal profile of the density and the physical parameters of the plasma. The speeds of ions, in the trapping region, remain practically constant for relatively long time intervals of the order of tens of nanoseconds. For the same initial conditions a magnetic field of higher value up to *B* = 200 T was used and the trapping time was improved. The conclusions of this investigation have shown the feasibility of high-energy ions interactions in the plasma for long trapping time, increasing the number of nuclear fusion reactions per laser shot. This choice allows the improvement of neutron production with moderate laser beam intensities.

3. The pulsed power configuration was investigated including the physical and geometrical parameters of capacitor banks and flat transmission lines respectively. Simulations on temporal evolution of pulsed high-density current and high magnetic field generated in a single (or double) turn metallic coil have been performed. Single and double turn metallic coils have been designed, using a commercial CAD code with specific geometrical parameters capable to support pulsed high-current densities and magnetic field pressure. The spatial dimensions of the double turn coil of the metallic structure are 3cmx2cmx2cm, the hole in the centre of the coil is 8mm and the thickness of the structure is 6mm. In addition, numerical tests have been done concerning the evaluation of two different commercial codes [ANSYS and “Maxwell”-ANSOFT] calculating magnetic field topology generated by a pulsed high current density in the single turn coil metallic structure.

4. A code module was developed including the nuclear fusion reactions in order to describe the temporal evolution of the species D, T, He and n in a fusion plasma. The cross

sections were calculated using analytical semi-empirical formulas from the literature, available for the kinetic energy range between 5keV and 200keV. The reaction rates depend strongly on the temporal and spatial evolution of the plasma density and temperature which were calculated using the 1.5_D MHD code. The module resolves simultaneously 4 time-dependent differential equations describing the involvement of the different species in each nuclear reaction as a function of the temperature and plasma density of the expanded plasma.