3.4 Theory and modelling

3.4.1 Stationary MHD modes in magnetically confined plasmas

Background and Objectives: This activity aims at constructing equilibria and relaxed states of laboratory plasmas of fusion concern (e.g., plasmas of tokamaks and reversed-field-pinches) with flow or/and finite electrical conductivity and investigating their linear and nonlinear stability. Understanding these issues can contribute to improving the current magnetic confinement systems and possibly developing new ones. kamagra brausetabletten viagra soft tabs argaiv1135

Work performed in year 2008 (in co-operation with IPP):

(i) The visco-resistive MHD equations with flow were studied in axisymmetric sytems by adopting a kinetic-viscosity term ($\nu\nabla^2 u$) in the momentum equation and an isotropic resistivity term in Ohm's law. In particular quasi-static flows were considered by including the velocity only in Ohm's law. It turns out that the equilibrium reduces to a set of partial differential equations which in general should be solved numerically. The "cat-eyes" steady state solution in the framework of hydrodynamics describing an infinite row of identical vortices was extended to the MHD equilibrium equation with incompressible flow of arbitrary direction. The extended solution covers a variety of equilibria including one- and two-dimensional generalised force-free and Harris-sheet configurations which are preferable to those usually employed as initial states in reconnection studies. Although the vortex shape is not affected by the magnetic field, the flow in conjunction with the equilibrium nonlinearity has a strong impact on isobaric surfaces by forming pressure islands located within the "cat-eyes" vortices (see **Annex 30**).

(ii) A recent sufficient condition for the linear stability of generic equilibria with incompressible flows parallel to the magnetic field [G. N. Throumoulopoulos, H. Tasso, Phys. Plasmas 14, 122104 (2007)] was applied to tokamak equilibria in cylindrical geometry. Both monotonically increasing and reversed magnetic shear safety factor profiles were considered. For cylindrical configurations and various flow profiles it turns out that the condition can be satisfied only for reversed magnetic shear equilibria in a thin internal layer of the poloidal cross section for large values of λ^2 lying on the border of accessible flows in tokamaks and large and positive values of $d\lambda^2/dr$ [$u = \lambda B$ with B in Alfvén velocity units]. For axisymmetric equilibria an analytic solution was constructed in terms of Coulomb wave functions with pressure gradient, toroidal current density and flow vanishing on the plasma boundary. For vanishing flow this reduces to the Hernegger-Maschke solution (see **Annex 31**). The aforementioned stability condition was applied to a) the Coulomb wave function solution for both regular configurations (with nested magnetic surfaces) as well as reversed current density configurations (with non nested magnetic surfaces) having nearly vanishing (average) toroidal current and b) to the extended cat-eyes equilibrium solution. In the former case, for ITER relevant parametric regions it turns out that the stability condition is satisfied in regions with low current density and large variation of the magnetic field perpendicular to the magnetic axis in connection with the Shafranov shift, irrespective of flow (see Annex 31). In the latter case (cat-eyes equilibrium), a magnetic-field-aligned flow of experimental fusion relevance and the flow shear have significant stabilising effects in the region of pressure islands. The stable region is enhanced by an external axial (“toroidal”) magnetic field. Comparison of the stability results in the linear and nonlinear regimes reveals the importance of equilibrium nonlinearlity in flow-caused stabilising effects. Details are given in Annex 30.

3.4.2 Turbulence and anomalous transport phenomena

Background and Objectives: Task (i) concerns the development of a reductive Lagrangian perturbation method for the investigation of turbulent transport in tokamak magnetic field environment. Task (ii) concerns the investigation on the feasibility of extending the Hamiltonian methods applied in the Vlasov-Maxwell system to multi-phase space correlation functions. First attempts for a unified description of collisional scattering and rf -induced quasilinear spatial and velocity space diffusion. Task (iii) concerns the investigation of the interaction of energetic ions with rf-waves and the associated instabilities. Task (iv) concerns the exploration of particular turbulence models that are based on the concept of Self-Organised Criticality (SOC). The turbulent plasma is considered as a complex system, in which localised instabilities are relaxed in small-scale diffusive events. A particular aim of our approach is not to make use of the usual sand-pile analogy, but to use the natural physical variables instead and to make the processes in the model mimic the processes that are believed to actually be active in the considered physical systems. We follow two approaches: (1) We develop the Extended Cellular Automaton (X-CA) model for the magnetic field, a fully MHD compatible SOC model that allows detailed and physically interpretable studies of turbulent magnetic fields in the state of SOC. (2) We construct a SOC model for ITG mode driven turbulence. Task (v) concerns the use of techniques for modelling the stochastic, turbulent, global structure of the magnetic field. The task includes the formulation of the magnetic field evolution in an appropriate Hamiltonian form, and the construction of a mapping with the use of a symplectic integration scheme. In parallel, alternatives for the computationally efficient modelling of stochastic magnetic fields are explored, including the direct numerical generation of self-consistent stochastic magnetic fields with prescribed statistical properties. Emphasis of this task is on the impact of the stochastic magnetic fields on transport. Task (vi) concerns the investigation of the Langevin and the corresponding Fokker-Planck equations describing the transport of charged particles, subject to inhomogeneous magnetic fields and to stochastic forces modelling the interaction of the particles with the bulk of the plasma. The proposed work will provide some insight to the transport behaviour of minority charged species in the core and of impurities coming from the wall. Finally, task (vii) concerns the development of Eulerian and Lagrangean CFD models for the study of near-wall motion of particulate matter and its interaction with stochastic, turbulent fluid flow in the presence of deterministic or stochastic magnetic fields.

Work performed in year 2008 (in co-operation with the Institutes indicated):

(i) A Lagrangian reductive perturbation method incorporating non-axisymmetric magnetic field perturbations (which are treated on the same order in the perturbation scheme with the applied magnetic field inhomogeneity) was employed. The density fluctuations considered at the plasma edge was used as a model for the density blobs known to exist in this region. These fluctuations were also considered in the same order as the rest in the perturbation scheme. The goal was to estimate the contribution of these phenomena in the transport. To this goal complex confining magnetic field topologies were considered as well as chaotic ones: In our formulation the charged particle dynamics at the microscopic level incorporated the aforementioned complex magnetic field topology (that is, islands, triangularity, etc) as well as spatio-temporally localised rf-fields and chaotic magnetic field perturbations. A particle orbit numerical integrator is under development. (In co-operation with PSFC-MIT.)

(ii) The Vlasov-Maxwell system is intrinsically nonlinear through the self-consistent fields in the plasma. For the electrostatic case, nonlinear distribution functions exist (in the sense that they considerably deviate from the well known Maxwellian) that account for the trapping of particles in the electrostatic potential which has been extensively studied in the past (BGK modes). The extension of these approaches in the presence of a homogeneous as well as inhomogeneous confining magnetic field under the influence of externally injected RF waves for relativistic electrons has been investigated. The extension of the Lie perturbation scheme for the Vlasov plasma to treat binary collisions in action space was also formulated in a preliminary fashion. (In co-operation with PSFC-MIT.)

(iii) The interaction of energetic ions with rf-waves and the associated instabilities was investigated: The interaction of two high frequency waves with a frequency beating around the ion cyclotron frequency has been considered. Resonant interaction was also incorporated. The presence of possible associated instabilities and ponderomotive effects driven by the spatial localisation of the rf-waves is under investigation. The penetration capability of these high frequency waves makes possible the energization of ions near the plasma core. (In co-operation with PSFC-MIT.)

(iv) Self-organised criticality (SOC): (1) Continuing work from the previous year, we further developed the X-CA model in order to improve the magnetic topology in the SOC state. The set-up was adjusted to the 2-dimensional poloidal cross-section, with assumed axisymmetry. The loading process was modified so that it models the generation of the poloidal field through the driving of the toroidal current, whereas the toroidal field remains unaltered. The instabilities are threshold-dependent on the current, and they are relaxed by local diffusion events. The system reaches the SOC state as a dynamic equilibrium, with a degree of magnetic fluctuations that is characteristic for reversed field pinch devices. Flux surfaces are calculated, and a high stiffness of the magnetic field configuration was found in off-axis driving set-ups. Test-particle simulations were not done yet in the period in subject, since higher priority was given to the development of the SOC model for ITG driven turbulence [see (2)], and to the test particle simulations described in (v). (For details see **Annex 32** .) (2) In addition to the X-CA model for the magnetic field, a second project was initiated, in which a SOC model for ITG mode driven turbulence was constructed. The model describes the evolution of the ion temperature, whereby the evolution rules follow closely the basic physics of the ITG mode instability and are formulated in terms of the natural variable of the system, the ion temperature. The appropriate instability criterion and the relaxation rules for the instability were determined, and it has been assessed that the system does reach the SOC state. The temperature profiles in SOC state exhibit very high stiffness, remaining unaffected by the loading pattern applied (central or off-axis heating). The temperature profiles are of exponential shape, and they are found to be in good agreement with those experimentally seen at JET. For details see **Annex 33**. (In co-operation with JET.)

(v) Techniques for modelling stochastic magnetic fields: (1) A new mapping for the safety factor and a canonically conjugate angle had been constructed from the tokamap in the previous year, by applying a suitable variable transformation. Further investigation of the new mapping in the period in subject revealed that the new mapping is in principle equivalent to the tokamap, the non-linearities lead though to a much faster loss of precision, which makes the approach through variable transformations in the tokamap-Hamiltonian inadequate for the application to the three magnetic field components, since three mappings would have to be constructed from the same Hamiltonian that must be expected to yield completely de-correlated magnetic field components. The tokamap Hamiltonian is actually a strongly simplified model for field-line transport, and it has only one degree of freedom. We therefore chose in a next step the more realistic and more complex guiding-centre Hamiltonian (with two degrees-of-freedom) as a starting point for the construction of the mapping. The continuous equations of motion for the guiding centre variables were derived, and, for possible non-canonical variable transformations, the relevant Poisson brackets were calculated. It turned then out that the construction of a mapping for the original guiding centre variables themselves is hardly feasible, due to the high complexity of the equations, so that any transformation to the magnetic field as primary, evolving variables is practically impossible. (2) As an alternative approach, stochastic magnetic and electric fields were directly generated numerically, with prescribed statistical properties. Starting from a given auto-correlation function, the fields are constructed in Fourier-space with the use of the Wiener-Khinchine theorem and by applying then a back-transformation to real space. The magnetic field is determined as the curl of the stochastic vector potential, whereto the derivatives are calculated again via Fourier space. Moreover, a new test-particle code was developed, based on the guiding-centre velocity approximation, which allows to trace particles (including impurities) in the 3D stochastic field environments and to determine their diffusive behaviour. In the period in subject, both numerical tools were completed and successfully tested. This work will be continued in the next period, where ion drift in stochastic magnetic fields will be studied, and the results will be compared to those yielded by the De-Correlation Trajectory (DCT) method. (In co-operation with ULB and MEdC.)

(vi) Stochastic motion due to a random force and/or electric field: A charged particle in a uniform stationary magnetic field, subject to an additive Gaussian random force has been studied. The Langevin equations governing the velocity and position of the particle are linear and can be solved exactly. In addition, the corresponding (exact) Fokker-Planck type equation for the probability density of the position and velocity of the particle has been established. Its fundamental solution and asymptotic properties have been derived. For coloured noise, a numerical scheme has been developed and sample paths of the particle have been displayed for various initial velocities and positions (see **Annex 34**). We have chosen to postpone the study of the random motion in inhomogeneous stationary magnetic fields, as it was initially planned, and instead we have worked with a computational scheme, based on the expansion of the random force as a Fourier series. This modification was necessary in order to investigate general random forces and fields. The implemented approach is considered as more general and promising because it allows the investigation of the random motion of the particle subject to a Gaussian force with an arbitrary covariance, extending therefore, the results obtained for "coloured" noise. Simulations for a Lorentzian covariance are being performed and the accuracy of this approach is being examined. (In co-operation with ULB and MedC.)

(vii) The development in the form of a computational fluid dynamics solver for viscous MHD flows that accounts for the motion of discrete particles in a Lagrangian reference frame was completed. A number of direct numerical simulations in MHD turbulent channel flow were performed in order to study complex particle/turbulence interactions, considering large ensembles of particles with different representative response times. The effect of direction and amplitude of the external magnetic field on the particle statistics (such as particle concentration, mean and rms velocities and higher moments) are studied. The level at which particle motion is affected from the underlying correlated fluid motions and how these are influenced by the external magnetic field are studied. For that reason, particle interactions with the near-wall turbulent coherent structures in the presence of a Lorentz force is being investigated. Finally, the effect of the magnetic field in the particle deposition rates at the solid walls is examined. For more details see **Annex 35** and Annex 36 .

3.4.3. Discrete kinetic models for transport

Background and Objectives: Discrete kinetic simulations may provide an alternative mesoscale type of description of the transport properties of complex physical systems. The main unknown is a particle distribution function, which obeys a suitably chosen kinetic equation, while the bulk quantities of practical interest are obtained by taking moments of the distribution function. The benefit of this description, compared to the more traditional micro and macroscopic approaches, is due to the fact that physics, at particle level, may be investigated, with moderate computational effort. This activity aims to develop kinetic computer codes capable to simulate in a computationally efficient manner non-linear resistive MHD flows and vacuum systems in fusion devices.

Work performed in year 2008 (in co-operation with the Institutes indicated):

(i) MHD flows using lattice kinetic (Boltzmann) schemes: The LK3D code has been extended to include non-uniform Cartesian grids. The grid may be refined in a manual and/or automatic (self consistent) manner by implementing the roots of orthogonal polynomials (e.g. Legendre, Chebyshev) for discretising the physical space. We have found that in order to achieve that, the discretisation in the physical space may not be directly connected with the discretisation in the molecular velocity space. This is crucial, since it provides more freedom to the choice of the discrete velocity set and also even more important it generates suitable non-uniform grids when it is needed as in the case of high Hartmann numbers. This approach has been applied with considerable success in one-dimensional flows (e.g. Hartmann flow) but its multi-dimensional version requires further development. (In co-operation with ENEA.)

(ii) Vacuum flows and systems in fusion devices: Most of the objectives set for 2008 have been successfully completed and some additional work not initially scheduled has been initiated. The detailed comparative study between the computational results based on linear kinetic theory obtained at UoThly and the corresponding experimental data obtained at the TRANSFLOW facility of FZK in the case of long channels of various cross sections has been continued for various flow configurations (see **Annex 37**). In addition to the computed flow rates, a methodology has been developed to estimate the pressure drop distributions as well as other characteristic parameters along the channels. Next, the “in house” Direct Simulation Monte Carlo (DSMC) probabilistic algorithm for simulating gas flows through orifices and channels of finite length has been further developed and upgraded (see **Annex 38**). In particular, different types of intermolecular potentials based on the inverse power law as well as the Cecignani-Lampis boundary conditions have been included, providing a more complete physical description. Also the grid refinement has been enhanced by including a three-level refinement grid supported by the so-called weighted zone process. Results obtained by the upgraded DSMC code are in very good agreement with corresponding experimental results available in the literature. In addition, we have participated in the corresponding measurements performed at FZK. The comparison between computations and experiments for flows through short channels, have been delayed due to certain pitfalls both in modelling and experiments and they will be performed next year. Finally, we have started working on the development of non-linear kinetic codes in order to have in addition to the DSMC approach an alternative computational tool for simulating flows in complex configurations including cryogenic pumps. These nonlinear kinetic codes have been already benchmarked by simulating accurately certain flows and transport phenomena (see **Annex 39**). It is important to note that the implemented analysis may be applied and the obtained results are valid in the whole range of the Knudsen number, i.e., under low, medium and high vacuum conditions, as is the case in the vacuum systems of DT fusion machines such as ITER and DEMO. (In co-operation with FZK.)

3.4.4 3D Spectral full MHD Code: Scalar and particle transport in MHD turbulence

Background and Objectives: Strong magnetic fields tend to suppress turbulence in the liquid metal circulating through the reactor blanket. However, curvature, system rotation, and fringing magnetic fields can, under certain conditions, destabilise the flow and lead to turbulence enhancement. Enhanced instabilities and turbulence can have a significant effect on the local heat transfer rates, leading to thermal localised gradients. It is therefore important to understand the combined effects of mean shear, strong magnetic fields and system rotation on heat transfer effects. The goal of this activity is to examine the transport of a passive scalar in the presence of strong magnetic fields, mean shear, and system rotation using a fully-3D spectral code. (This is a multi-annual activity performed in cooperation with ULB).

Work performed in year 2008:

(i) The series of direct numerical simulations (DNS) of passive scalar transport that was initiated in 2007 was completed in 2008 with the addition of several cases at higher hydrodynamic Reynolds numbers than those used in 2007. The new simulations became possible as a result of an upgrade in the computational system.

A second set of simulations, employing different combinations of dimensionless parameters than those used in the 2007 simulations, led to unexpected trends in the observed levels of scalar fluxes. The results of the new 2008 simulations were analysed and compared to the results of the 2007 simulations. This resulted in a better understanding of the role played by the structure dimensionality of the turbulence in passive scalar transport and led to the introduction of the passive scalar structure-dimensionality concept. This concept was further extended to describe the clustering of a dispersed phase in MHD turbulence.

The analysis of the 2007 and 2008 simulations was completed and the implementation of Lagrangian particle transport in 3D spectral full MHD code was initiated. Preliminary results were obtained for the case of non-sheared turbulence, while code modifications that will allow the simulation of Lagrangian particle dispersion under the combined effects of mean shear and rotation will be completed in the 2009.

3.4.5 2-D MHD code for plasma trapping in fusion devices including specific modules on magnetic field topology, neutronics and neutron-pellet interaction

Background and Objectives: The development of a two-dimensional MHD code in cylindrical coordinates is investigated in order to study the temporal and spatial evolution of high density and temperature plasmas in a compact fusion device with mirror-like magnetic topologies. The code can handle very large magnetic fields and very steep gradients of the plasma parameters. The interaction of a high intensity ultra-short laser pulse with a molecular beam of neutral deuterium clusters produce the high density and temperature plasma which expands very fast in vacuum decreasing both the local density and the number of D-D ions nuclear fusion reactions. The application of an external high magnetic field enables to decreases the plasma expansion velocity, increases the trapping time of the plasma and improves the neutron production. During the laser-plasma interaction the plasma volume is described, in first approximation, by a double density layer with different plasma densities. The first layer concern the high-density and temperature plasma produced by the pulsed laser beam and the second the lower density one (at least two order of magnitude) due to the tails of the laser beam profile. The high density plasma from the first layer expands into the second low-density plasma forming a shock wave. This expansion effect is available for both directions the radial which is perpendicular to the external applied magnetic field and the axial parallel to the magnetic field. The novelty of our numerical simulation is based on the introduction of the shock tube model in order to describe correctly the particular initial spatial and temporal conditions of the laser plasma production, the temporal evolution of the plasma parameters and the trapping time in the external applied magnetic field. The numerical model simulates the initial plasma conditions by introducing a step function describing the double plasma density layer with different spatial occupation for the radial and the axial direction (z-direction). The laser-plasma interaction produces plasma density up to 1019 cm-3 and accelerates D ions to high kinetic energy up to 50 – 70 keV. The objectives are: (1) the development of the two-dimensional MHD code in axisymmetric cylindrical coordinates with a high external applied magnetic field, (2) to describe initial plasma conditions for the simulation in order to maintain the high density plasma in the interaction volume as long as possible and decrease the plasma expansion velocity for various external applied magnetic fields and (3) investigate studies on a new configuration for high magnetic field generation using pulsed power techniques. For the numerical calculation the energy of the produced ions will fixed to 50 keV which is in good agreement with experimental results and the value of the magnetic field will vary from 50-150 Teslas which can be achieved experimentally using high voltage capacitor banks and spark-gaps.

Work performed in year 2008 (in co-operation with Ecole-Polytechnique-Paris and CEA):

(i) 2-D MHD code: The development of the two-dimensional MHD code (2-D MHD code) was investigated as was scheduled initially after the results of the 1.5-D MHD code at the end of 2007 (see **Annex 40**). The work is oriented to describe the expansion of a high density and temperature plasma in a compact fusion device using a resistive 2-D MHD code in axisymmetric cylindrical geometry. The code composed by a number of modules that solve the conservation equation for mass, momentum and energy of plasmas coupled with the external magnetic field equations. In the first part of the present work we include the solution of MHD equations in cylindrical coordinates based on the shock tube model in order to describe the initial conditions of the plasma formation in the high external magnetic field. The output from the different modules describes the temporal and spatial evolution of the physical parameters of the plasma such as density, pressure, temperature, expansion velocity, trapping time and magnetic field. For the period in subject, the modules are used to describe similar physical process, as was the case for the 1.5-D MHD (see **Annex 40**), in order to test every module and compare the new numerical results with the previous. Different tests were performed concerning the stability of the numerical simulation for relatively long trapping time up to 100 nsec and high external applied magnetic field up to 150 Tesla. In the second part of the numerical simulations a module was added describing the topology of the external magnetic field in the mirror-like configuration. The description is based on semi-analytical expressions, which are valid only as a first approximation; however, this approach is necessary in order to test the results and the coupling of the two parts of the code. The work will be continue to improve the code modules describing the physical parameters and the process of the high density, high temperature plasma and the coupling with the external applied magnetic field. The results are in agreement with existing experimental results.

(ii) Rates of nuclear fusion reactions: The development of a new numerical module is investigated. The work for the first period concerns the collection of the data and the semi-empirical formulas regarding the cross section of the main nuclear fusion reactions occurring during the plasma evolution. The module describe the temporal evolution of the products from 6-8, nuclear fusion reactions occurring in the high density high temperature plasma of a compact fusion device with external applied magnetic field. The two last nuclear fusion reactions referred to the interaction for T production from Li. A series of tests and verification was performed to verify the stability of the simulation for relatively long plasma trapping time.

(iii) Investigation on high magnetic field generation During the previous year (2007) we have investigate the production of high pulsed magnetic field produced by the discharge of high voltage capacitor-bank in a single turn coil in a mirror like configuration. A flat transmission line was proposed for the transfer of the stored electric energy in the capacitor bank to the coil. For the coupling between the flat line and the coil the choice of a multi-channel spark-gap enables to decrease the inductance of the equivalent circuit and improve the pulsed current in the coil. The results of our investigation show that the proposed configuration allows the development of a relatively compact generator for high magnetic field up to 70 Tesla. The extrapolation of the capacitor bank voltage to 50 kV shows that a magnetic field up to 90 Tesla can be achieved. But the numerical simulation from the developed MHD code show that for high neutron production longer trapping time of the plasma and higher magnetic field up to 150-180 Tesla are necessary. Under these conditions at least 3 modules of the proposed configuration are necessary to achieve the request value of the magnetic field. The realization of such a configuration is possible if the three spark-gaps connecting each capacitor bank with the single turn coil can work without electrical delay between them in order to transmit the maximum current from each capacitor bank to the coil. This condition is very difficult to achieve for future experiments and is expensive. A new solution was proposed concerning the design and the calculation of a new spark-gap. The advantage of the proposed new configuration enables the use of a certain number of capacitor bank modules with only one spark gap. The discharge in the gap of the proposed spark gap will be guided by a series of parallel lines and each line will contain a number of metallic spheres (~1cm in diameter) separated by a certain distance between them. The novelty of the proposed configuration is to use the new spark gap in flat or cylindrical configuration without main modification. A cylindrical configuration will facilitate the vacuum transition and the coupling of the proposed configuration with an experimental chamber.